Physics with Calculus 1 Prep Workshop

Conversions

Whenever you measure something, you measure it in some **unit**. A unit helps to describe and give context to a measurement. People would be confused if you say your car weighs 1, but if instead you say it weighs 1 ton then that measurement has meaning. In physics, it is of vital importance to understand units, and to be able to switch between them. Here are some common conversions, and a list of metric prefixes.

Length:

1 foot	=	12 inches
1 yard	=	3 feet
1 mile	=	5,280 feet
1 inch	=	2.54 centimeters
1 meter	=	3.281 feet
1 mile	=	1609 meters

Weight (on Earth):

1 pound (lb)	=	16 ounces (oz)
1 pound (lb)	=	4.448 Newtons (N)
1 ton	=	2000 pounds

Volume:

1 cubic meter (m^3)	=	1000 liters
1 gallon	=	3.786 liters
1 fluid ounce	=	29.57 cm^3
1 quart	=	2 pints
1 gallon	=	4 quarts

Time:

1 minute	=	60 seconds
1 hour	=	60 minutes
1 day	=	24 hours
1 week	=	7 days
1 month	=	30 days = 4 weeks
1 year	=	365 days = 52 weeks
1 decade	=	10 years
1 century	=	100 years

Metric Prefixes:

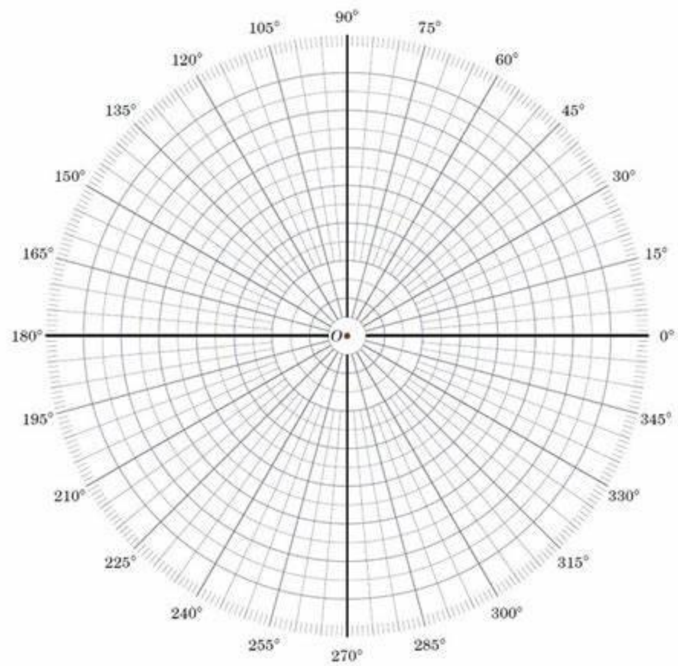
Prefix	**Symbol**	**Exponent**
Peta	Z	10^{15}
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
Kilo	k	10^{3}
Hecto	h	10^{2}
Deca	da	10^{1}
(No Prefix)		1
Deci	d	10^{-1}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	µ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}

Sample Conversion Question: Convert 3.00 decameters per second to miles per hour.

$$\frac{3.00\ decameters}{1\ second} \times \frac{10\ meters}{1\ decameter} \times \frac{1\ mile}{1609\ meters} \times \frac{60\ seconds}{1\ minute} \times \frac{60\ minutes}{1\ hour} = 67.122\ \frac{miles}{hour}$$

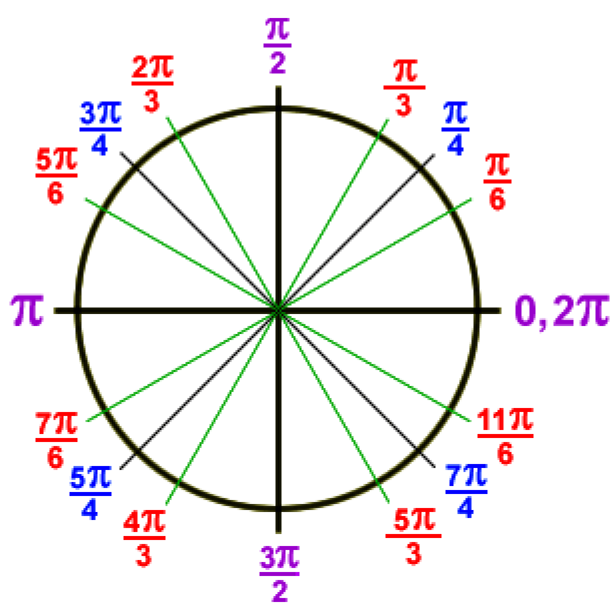
Trigonometry Concepts

Trigonometry, at its core, deals with angles. An angle is a measure of the difference in direction of two lines. Angles can be measured in degrees or radians. To measure in degrees, a whole rotation is called 360 degrees (or 360°). An example of the unit circle in degrees is provided below.



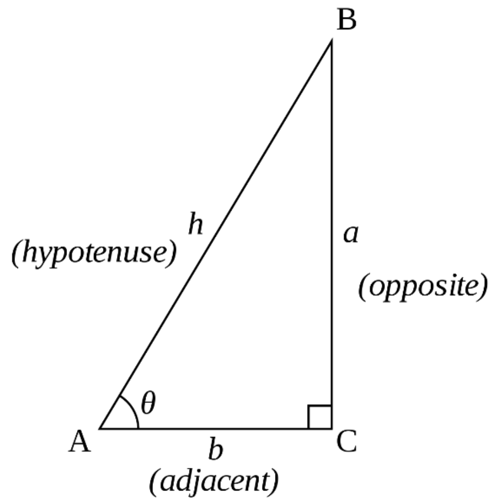
A more useful way to measure angles for physics, however, is radians. In radians, a whole rotation is measured as 2π rather than 360°. To convert from degrees to radians, you simply have to multiply by the conversion factor $\frac{\pi}{180°}$. To convert from Radians to degrees, you can multiply by $\frac{180°}{\pi}$.

Note: Since 2π = 360° then π = 180°



Sin, Cosine, and Tangent

While we can understand angles and distances separately, how can we understand the relationship between the two? The answer is simply trigonometric functions. These functions are easiest understood with a right triangle.



Here, θ refers to the angle we are currently considering, and the terms "Opposite" and "Adjacent" are placed with relation to this angle (opposite is across from the angle, whereas adjacent is directly next to the angle). The corner with the box (opposite the hypotenuse) represents the right angle.

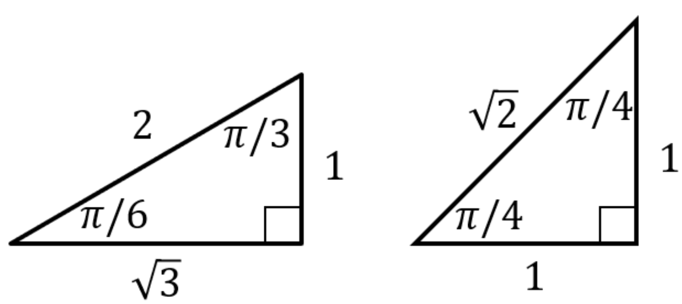
Given this image, we create the following relations:

Sin(θ) = $\frac{Opposite}{Hypotenuse}$; (Sin(θ) is pronounced "Sign of They-Tah")

Cos(θ) = $\frac{Adjacent}{Hypotenuse}$; (Cos(θ) is pronounced "Coh-sign of They-Tah")

Tan(θ) = $\frac{Sin(\theta)}{Cos(\theta)}$ = $\frac{Opposite}{Adjacent}$; (Tan(θ) is pronounced "Tan-jent of They-Tah")

With these measures, we can compute the relationship between distances and angles. To make it easier, we tend to remember 2 triangles:



From here, all of the trigonometric functions of these common angles can be found with ease.

Sample Trigonometric Function Question: Find the $\sin\left(\frac{\pi}{3}\right)$.

$$\sin\left(\frac{\pi}{3}\right) = \frac{Opposite}{Hypotenuse} = \frac{\sqrt{3}}{2}$$

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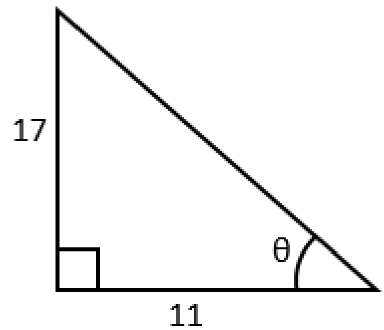
Other Trig Tricks

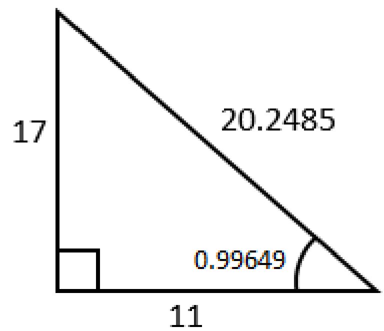
We know we can apply a trig function to an angle and get a ratio of sides… but what if a ratio of sides was given, and we need an angle? For example, if we have the below listed triangle



We could find the angle by taking an inverse trig function. For instance, just like

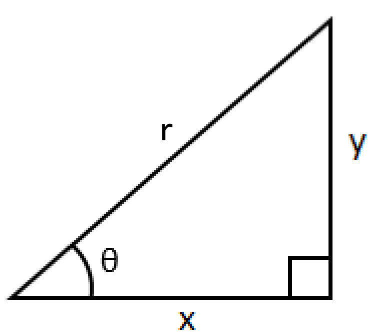
$\tan\theta = \frac{17}{11}$, we can say $\tan^{-1}\frac{17}{11} = \theta$, which gives us $\theta = 0.99649$ radians.

Let's say we wanted to find the measure of the hypotenuse of this same triangle. Since we know two of the sides, we can find the hypotenuse with the Pythagorean Theorem ($c^2 = a^2 + b^2$). In other words, we find $c^2 = (17)^2 + (11)^2 = 289 + 121$, so therefore $c = \sqrt{289 + 121} = \sqrt{410} \approx 20.2485$.



Other things to note:

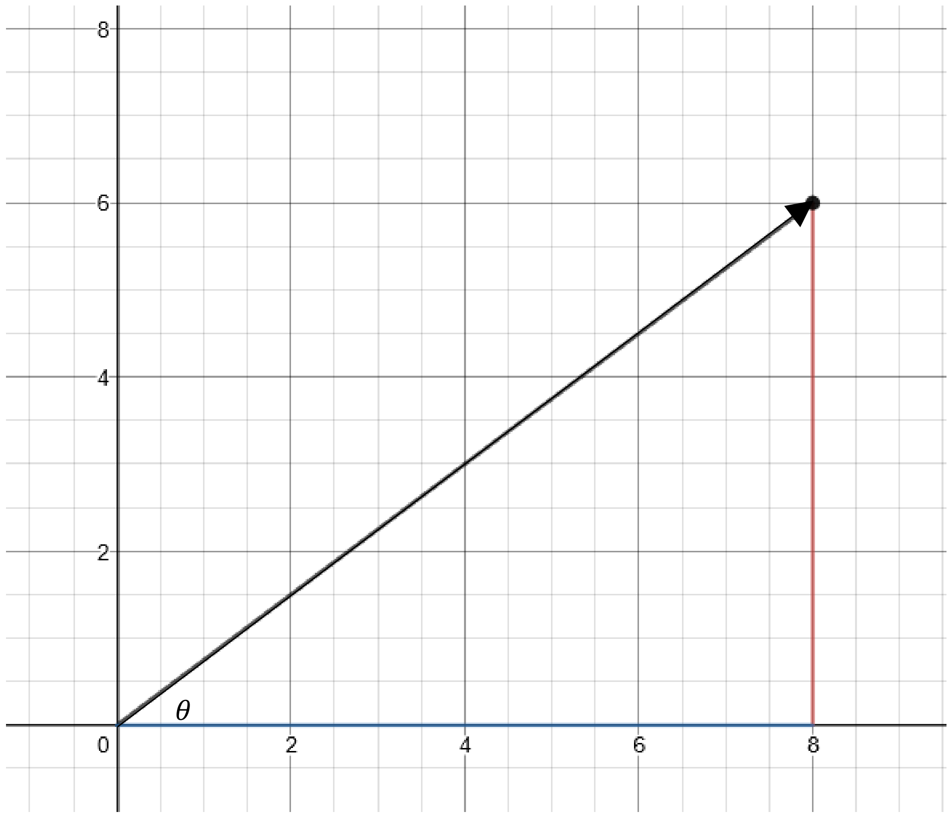
In the triangle depicted below



Where r is the hypotenuse, notice that $x = r\cos\theta$, $y = r\sin\theta$, and $\theta = \tan^{-1}\frac{y}{x}$

Vectors

There is a big difference between scalar and vector quantities. Scalar quantities are numbers, units, things that have no direction. Vectors, however, are much different. Many times when we are dealing with physical forces, there are two elements to that force – magnitude and direction. Magnitude tells us how much, and direction tells us where. For example, the vector <8, 6> tells us that we are moving a distance 8 in the x direction, and a distance 6 in the y direction.



Often, we want to figure out our direction and magnitude. The magnitude of the vector is the same as the hypotenuse of this right triangle we have made. Thus, to find the magnitude, we would compute

$$\sqrt{8^2+6^2}=\sqrt{64+36}=10$$

To find the angle, we can use inverse trig functions, such as the inverse tangent.

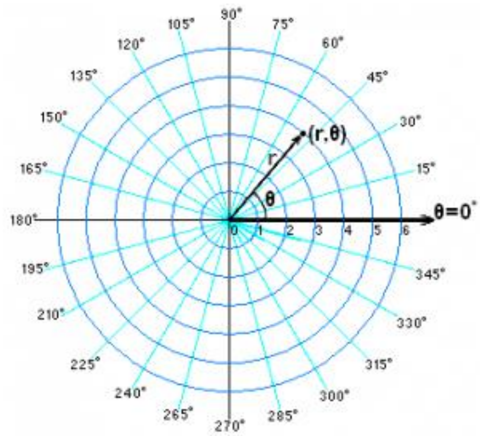
$$\theta=\tan^{-1}\frac{y}{x}=\tan^{-1}\left(\frac{6}{8}\right)\approx 0.6435\,radians$$

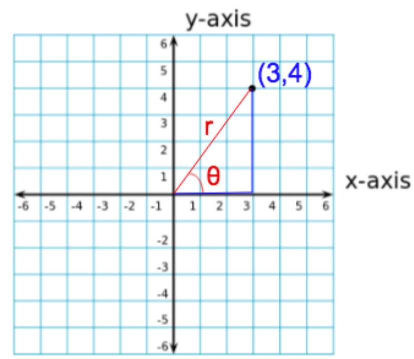
Therefore, we could express the vector

$<8{,}6>$ as $10<\cos(0.6435)\,,\sin(0.6435)>$, or $<10\cos(0.6435)\,,10\sin(0.6435)>$

Polar Coordinates

Polar coordinates are often used to describe vectors on the polar plane. The polar plane consists of rays (much like a vector) that emanate from an *origin* or *reference point*, with expanding concentric circles about the origin. This way, magnitude of a vector can be described via the term *r* for radius, which is the distance from the origin to the rim of one of the concentric circles. The direction of a vector is then described via the term θ for theta, which is the angle (in degrees or radians) formed between the ray emanating from the origin and the 0° axis (due east). As such, vectors and points on the polar plane are described with the coordinate (r,θ) rather than (x,y) used for Cartesian or rectangular coordinates.





Polar Coordinates (r,θ)
Cartesian Coordinates (x,y)

Let's say we have the Cartesian coordinates of a point in the x-y plane

$$(x, y) \ = \ (3.0, 4.0)$$

To find the polar coordinates, we must convert its x and y components to its radial coordinate (*r*) and angular coordinate (θ).

For *r* we use

$$r = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5.0$$

For θ we use

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{4}{3}\right) \approx 0.92730\, radians \text{ or } 53.13°$$

$$(r, \theta) \ = \ (5.0, 53.13°)$$

Converting *back* to rectangular coordinates we get

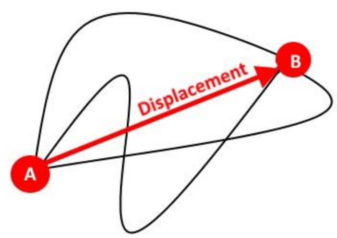
$$x = r\cos\theta = (5)\cos(53.13°) = 3.0$$

$$y = r\sin\theta = (5)\sin(53.13°) = 4.0$$

$$(x, y) \ = \ (3.0, 4.0)$$

Distance and Displacement

Distance and displacement are two terms used to describe the motion of an object in space. Travelled distance is a scalar quantity that refers to "how much ground an object has covered" during its motion. Displacement ($\overrightarrow{\Delta x}$) is a vector which begins at the starting point of motion and ends at the final position. Displacement represents net movement (magnitude and direction).



Travelled distance as the length of the path taken and displacement as the change between two positions.

As objects move, they *typically* undergo a change in position from one position to another. This change in position, or its *displacement*, is denoted with the symbols such as $\overrightarrow{\Delta x}$ or $\overrightarrow{\Delta y}$. The symbol Δ (pronounced "delta") denotes a change in any physical quantity. With this, you can describe the *position* of an object in space as it moves from an initial starting position $\overrightarrow{x_\iota}$ ("$ex\ initial$") to an end position $\overrightarrow{x_f}$ ("$ex\ final$").

$$\text{Displacement } \overrightarrow{\Delta x} = \overrightarrow{x_f} - \overrightarrow{x_\iota}$$

For example, a person at a point labeled A walks 3 meters north to a point B, then 4 meters east to point C, then 3 meters south to a point D, and lastly 4 meters west to stop back at point A.

To find the distance travelled by the person, we add.

$$\text{Travelled distance} = 3m + 4m + 3m + 4m = 14m$$

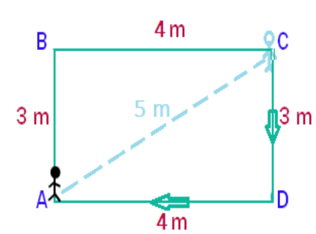
To find the displacement of the person, we subtract our initial position from our final position using point A as a reference.

$$\text{Displacement } \overrightarrow{\Delta x} = \overrightarrow{x_{f,A}} - \overrightarrow{x_{\iota,A}} = 0$$

If the person starts at a point C, but then moves 3 meters south to a new point D and 4 m west to a new point A, they would have travelled 7 meters instead and be displaced 5 meters from where they started.

$$\text{Travelled distance} = 3m + 4m = 7m \text{ , Distance from C to A } (\Delta d) = \sqrt{3^2 + 4^2} = 5,$$

$$\text{Displacement } \left(\overrightarrow{\Delta x}\right) = \overrightarrow{x_{f,A}} - \overrightarrow{x_{\iota,C}} \text{ .}$$



Because displacement is a vector, it may be expressed as having a positive magnitude and a direction, or it may be a negative magnitude if there is a point of reference such that the negative magnitude makes it clear which direction the displacement was in. For displacement, only the starting and ending positions matter.

Speed and Velocity

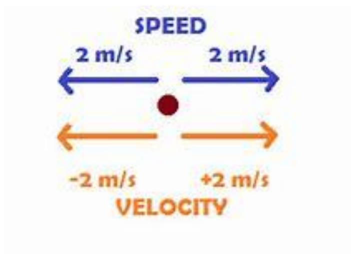
Speed and velocity are another two terms used to describe motion of an object in space. Speed $|\boldsymbol{v}|$ is a scalar quantity that refers to the *rate of change of distance*. It describes the magnitude of distance traveled per unit of time without regard to direction. Velocity (usually labeled v) is a vector that refers to an object's *rate of change of displacement*. Using velocity, you can describe the displacement of an object per unit of time.

$$Speed = \frac{distance}{time} \; or \; |\boldsymbol{v}| = \frac{\Delta d}{\Delta t}\text{; if the speed is constant}$$

$$Velocity = \frac{displacement}{time} or \; v = \frac{\overrightarrow{\Delta x}}{\Delta t}\text{; if the velocity is constant}$$

Speed $|\boldsymbol{v}|$ is essentially *distance over time* while velocity v is *displacement over time*. Since speed does not regard direction of motion, it is always described as a positive quantity (just like the scalar quantity distance). On the other hand, velocity can be described as both a positive and negative quantity since it describes the vector displacement over time.

For example, an object moving with a speed of 2 m/s would be moving 2 meters per second regardless of direction. Taking direction into account, it would have a velocity of 2 m/s to the *right* if positive, and 2 m/s to the *left* if negative.

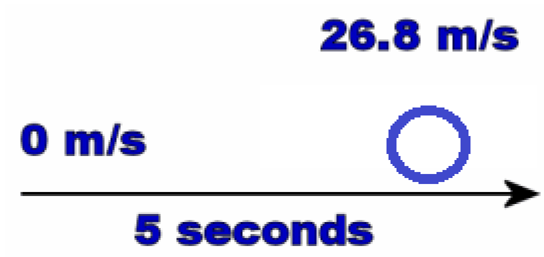


Acceleration

As an object is moving, it can gain and lose speed over the course of its motion. Acceleration (usually denoted as a) is vector that refers to the *rate of change in velocity*. It describes the magnitude of the change in velocity and the direction (positive or negative) of that change over a time interval. Simply put, acceleration is *rate of change in velocity over time*, and can be given as

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \text{ ; if } a \text{ is constant}$$

For example, an object at rest ($v_i = 0\frac{m}{s}$) *accelerates* uniformly up to a speed of 26.8 m/s in 5 seconds.

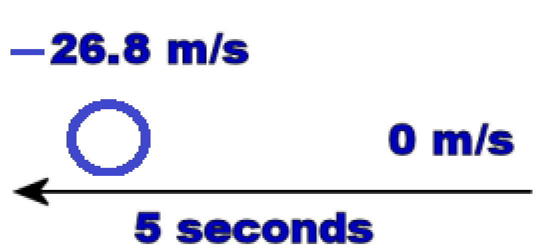


To find the acceleration, we divide the change in velocity by the change in time.

$$\text{If } v_i = 0\frac{m}{s} \quad v_f = 26.8\frac{m}{s} \quad \Delta t = 5s - 0s\text{, then}$$

$$a = \frac{26.8\frac{m}{s} - 0\frac{m}{s}}{5s - 0s} = \frac{26.8\frac{m}{s}}{5s} = 5.36\frac{m}{s^2}$$

If the same object accelerates in the opposite direction instead, its velocity would *increase negatively* as its increasing in speed toward the opposite direction. Its acceleration, likewise, would be negative



$$a = \frac{-26.8\frac{m}{s} - 0\frac{m}{s}}{5s - 0s} = \frac{-26.8\frac{m}{s}}{5s} = -5.36\frac{m}{s^2}$$

Laws of Motion (when a is constant)

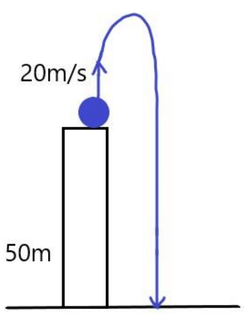
List of terms:

Δx $= displacement\ (distance\ moved)\ in\ a\ certain\ direction$
v_i $= initial\ velocity\ (original\ velocity)$
v_f $= final\ velocity\ (ending\ velocity)$
a $= acceleration$
t $= time$

Formulae: When a is constant,

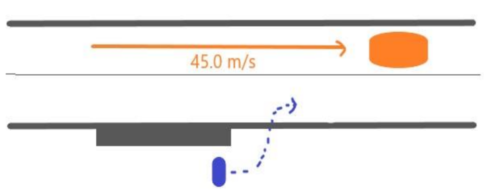
$$\Delta x = v_i t + \frac{1}{2}at^2$$
$$\Delta x = \frac{1}{2}(v_i + v_f)t$$
$$(v_f)^2 = (v_i)^2 + 2a\Delta x$$
$$v_f = v_i + at$$

Sample Linear Motion Question: A ball is thrown upward from the top of a building. It is given an initial velocity of 20 m/s upward. The building is 50 meters high, and the ball misses the edge of the building on its way down. How long does it take for the ball to achieve its maximum height? What is that height? (Assume that we are on the planet earth, where gravitational acceleration is -9.8 m/s^2)



When the ball is at its maximum height, $v_f = 0$. We know $v_i = 20\frac{m}{s}$, and $a = -9.8\frac{m}{s^2}$, and we are looking for Δx and t.
$v_f = v_i + at$, so $0 = 20 + (-9.8)t$, or $t = 2.041\ seconds$
$(v_f)^2 = (v_i)^2 + 2a\Delta x$, so $(0)^2 = (20)^2 + 2(-9.8)\Delta x$, or in other words $\Delta x = 20.41m$. Note that this is how high it goes above the building, so the total maximum height is $70.41m$.

Sample Linear Motion Question: A speeding car traveling at a constant speed of 45.0 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 6m/s^2. How long does it take the trooper to overtake the car?



From the information given, we know

$v_{car} = 45\frac{m}{s}$ $\quad v_{i,trooper} = 0\frac{m}{s}$ $\quad a_{trooper} = 6\frac{m}{s^2}$

Also, the car is given a one second lead

$\Delta x_{i,car} = (45m - 0m) = +45m\ head\ start$

Using the billboard as our origin, we can build an equation for both the car and the trooper to model their positions in respect to time t.

Note: The car is going at constant velocity (so, $a_{car} = 0\frac{m}{s^2}$)

$$\Delta x_{car} = v_i t + \frac{1}{2} a_{car} t^2 + \Delta x_{i,car} = \left(45\frac{m}{s}\right) t + 45m$$

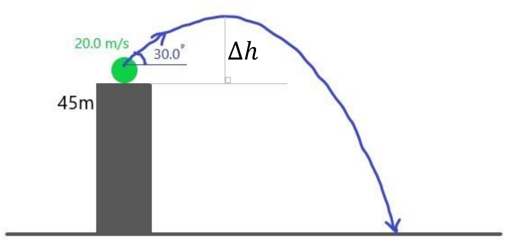
The equation of the trooper is $\Delta x_{trooper} = v_{i,trooper} t + \frac{1}{2} a_{trooper} t^2 = \frac{1}{2}\left(6\frac{m}{s^2}\right) t^2$

The trooper will successfully catch up to the car at a time t when both their positions meet. To solve for this t, we simply set their position equations equal to each other

$$\Delta \boldsymbol{x_{car}} = \left(45\frac{m}{s}\right) t + 45m = \frac{1}{2}\left(6\frac{m}{s^2}\right) t^2 = \Delta \boldsymbol{x_{trooper}}$$

$$0 = 3t^2 - 45t - 45, \quad t = 15.94s$$

Sample Linear (2-D) Motion Question: A water balloon is thrown upward from the top of a building at an angle of 30.0° to the horizontal and with an initial speed of 20.0 m/s. The point of release is 45.0 m above the ground. 1) How long does it take for the balloon to hit the ground? 2) What is the balloon's speed of impact? 3) How far away does the balloon travel from its initial position?



1)The water balloon will hit the ground when its *flight time*, t, is finished. So, we must solve for t; the amount of time the balloon is traveling in the air. We know it is thrown from an initial height (y_0) of 45.0 m, with an initial speed of 20.0 m/s, at an angle (θ) of 30.0°. So:

y_0 = 45.0 m, y_1 = 0 m

initial speed= 20.0 m/s, θ = 30.0°

After the water balloon reaches to the maximum height, it will fall to the ground. At the top (max height), $v_{top,y} = 0$.

$$v_{top,y} = v_{i,y} + at_{top}$$

Where $v_{i,y} = v_i sin\theta = \left(20.0\frac{m}{s}\right)\sin(30.0°) = 10.0\frac{m}{s}$. So,

$$0 = 10.0\frac{m}{s} + \left(-9.8\frac{m}{s^2}\right) t_{top}$$

$$t_{top} = 1.02\ s$$

Let Δh be the height from the top of the building to the max height.

For motion in the vertical axis to the max height from the building, our equation is:

$$\Delta h = v_{i,y} t_{top} + \frac{1}{2} a t_{top}^2$$

$$\Delta h = \left(10.0\frac{m}{s}\right)(1.02s) + \frac{1}{2}\left(-9.8\frac{m}{s^2}\right)(1.02s)^2 = 5.10m$$

Now, the balloon falls from the top ($45m + 5.10m = 50.1m$) to the ground with the initial speed at top, $v_{top,y} = 0$.

$$\Delta y = v_{top,y} t_{fall} + \frac{1}{2} a t_{fall}^2 = -50.10m$$

$$\Delta y = \frac{1}{2}\left(-9.8\frac{m}{s^2}\right) t_{fall}^2 = -50.10m$$

$$t_{fall} = 3.20s$$

So, the total time for the balloon to hit the ground is $1.02\ s + 3.20\ s = 4.22\ s$.

2) The water balloon's speed of impact is equal to the magnitude of its velocity, v_f, at impact. That is:

$$v_f = \sqrt{\left(v_{f,x}\right)^2 + \left(v_{f,y}\right)^2}$$

$$where\ \ v_{f,x} = \left(20.0\frac{m}{s}\right)\cos(30.0°) = 17.32\frac{m}{s},$$

$$v_{f,y} = \left(20.0\frac{m}{s}\right)\sin(30.0°) - \left(9.8\frac{m}{s^2}\right)(4.22\ s) = -31.36\frac{m}{s}$$

So we get $v_f = \sqrt{\left(17.32\frac{m}{s}\right)^2 + \left(-31.36\frac{m}{s}\right)^2} = 35.83\frac{m}{s}$

3) The horizontal displacement is based on the horizontal speed of the balloon and its flight time. This leads to the equation:

$\Delta x = v_{i,x} t + \frac{1}{2} a t^2$ (no horizontal acceleration, $a = 0$)

$$\Delta x = \left(17.32\frac{m}{s}\right)(4.22\ s) = 73.09\ m$$

System of Equations on a TI Graphing Calculator

If you have a system of equations:

$$\begin{cases} a_1x + b_1y + c_1z = S_1 \\ a_2x + b_2y + c_2z = S_2 \\ a_3x + b_3y + c_3z = S_3 \end{cases}$$

where a, b, and c represent coefficients of x, y, and z, and S represents the solution to the equation. To solve this on your calculator, go to the matrix menu (by pressing the 2^{nd} button, and then pressing the x^{-1} button). You will see a list of names for matrixes, such as [A], [B], and so on. Move two to the right to get to the "Edit" menu. Press Enter on the matrix name you wish to use. Next, alter the dimensions of the matrix so that your calculator recognizes it as a "3 x 4" matrix. For the first row, put in a_1 b_1 c_1 and S_1 (which, as we discussed before, stand for the coefficients in your system of equations). For the second row, put a_2 through S_2, and for the third enter a_3 through S_3. Your matrix should look like this:

$$\begin{bmatrix} a_1 & b_1 & c_1 & S_1 \\ a_2 & b_2 & c_2 & S_2 \\ a_3 & b_3 & c_3 & S_3 \end{bmatrix}$$

Now, press 2^{nd} and Mode to exit. Now, again go to the matrix menu by pressing 2^{nd} and then x^{-1}. Move to the Math submenu, and move down your list of options until you see "rref(" as an option. Click rref(. Next, go back to the matrix menu, click Enter on the name of the matrix you used before, and close the parenthesis. Press enter, and it will return an answer like this:

$$\begin{bmatrix} 1 & 0 & 0 & A_1 \\ 0 & 1 & 0 & A_2 \\ 0 & 0 & 1 & A_3 \end{bmatrix}$$

This means that $x = A_1$, $y = A_2$, $z = A_3$.

Questions

Conversion

a. If a car is traveling at a speed of 28.0 m/s, is it exceeding a speed limit of 55.0 mi/h?

b. The speed of light is about 3.00×10^8 m/s. Convert this to miles per hour.

c. An acre-ft is a unit of *volume* that covers an area of *one acre to a depth of one foot*. If an acre is 43560 ft^2, find the volume, in m^3, of a water reservoir containing 25.0 acre-ft of water.

d. A quart of ice cream is to be made in the form of a cube. What should be the length of a side, in cm?

e. The base of a pyramid covers an area of 13.0 acres (1 acre = 43560ft^2), and has a height of 481 ft. If the volume of a pyramid is given by the expression V=bh/3, find the volume of the pyramid in m^3.

f. The radius of the planet Saturn is 5.85×10^7m and its mass is 5.68×10^{26}kg. Find its density in g/cm^3. Also find the surface area of Saturn in ft^2. Note $V_{sphere}=4/3\pi r^3$ and $SA_{sphere}=4\pi r^2$

Trigonometry

a. A surveyor measures the distance across a straight river by the following method: Staring directly across from a tree on the opposite banh, he walks 100 m along the riverbank to establish a baseline. Then he sights across to the same tree. The angle from his baseline to the tree is 35.0°. How wide is the river?

b. A ladder 9.00 m long leans against the side of a building. If the ladder is inclined at an angle of 75.0° to the horizontal, what is the horizontal distance from the bottom of the ladder to the building?

c. In a certain right triangle, the two sides that are perpendicular to each other are 5.00 m and 7.00 m long. What is the length of the third side of the triangle?

d. A point is located in a polar coordinate system by the coordinates r = 2.5 m and θ = 35°. Find the x and y coordinates of this point in a similar x-y plane about the same origin.

e. Two points in a rectangular coordinate system have the coordinates (5.0, 3.0) and (-3.0, 4.0). Determine the distance between these two points.

f. If you have an origin point at (0.0, 0.0), determine the angle formed between a point at (5.0, 3.0) and a point at (-3.0, 4.0) with respect to the origin. Hint: draw these points on an x-y plane

Motion in one dimension

a. In 1865, Jules Verne proposed sending men to the Moon by firing a space capsule from a 220m long cannon. These theoretical astronauts would achieve a velocity of 10.97 km/s by the time they exited the cannon. (a) What would be the acceleration these astronauts would achieve during their time in the cannon? (b) Human beings can only survive accelerations up to 15g. Is Jules Verne's space cannon viable? Recall that g = 9.8 m/s^2.

b. A truck on a straight road starts from rest and accelerates at 2.0 m/s^2 until it reaches a speed of 20 m/s. Then the truck travels for 20 s at a constant speed until the brakes are applied, stopping the truck with a constant negative acceleration. It takes 5 seconds for the truck to fully come to a stop. (a) How long is the truck in motion? (b) What distance does it cover?

c. A train 400 m long is moving on a straight track with a speed of 82.4 km/hr. The engineer applies the brakes as the front of the train arrives at a crossing. The train clears the crossing, and the speed of the train as the back of the train clears the crossing is 16.4 km/hr. Assuming constant acceleration, determine how long the train blocked the crossing.

d. A ball is thrown upward vertically, and is caught by the thrower after 2.00 s. Find (a) the initial velocity of the ball, and (b) the maximum height the ball reaches.

e. A small mailbag is released from a helicopter that is descending steadily at 1.50 m/s. After 2.00 seconds, (a) what is the speed of the mailbag, and (b) how far is it below the helicopter? (c) Solve for the same questions, assuming that the helicopter is instead *rising* at 1.50 m/s.

Motion in two dimensions

a. An airplane flies 200 km due west from city A to city B, and then 300 km in the direction 30.0° north of west from city B to city C. (a) If you draw a straight line from city A to city C, what is the distance? (b) Relative to city A, in what direction does city C lie?

b. Long John Copper, a pirate, has buried his treasure on an island. His map , based on the cardinal directions, says to go 40 degrees North of West for 400m, then 15 degrees West of South for 100m, then 30 degrees North of East for 900m, and then due south for 100m. Answer the following questions:

   (a) What are the coordinates of the point where the pirate's treasure is buried? (b) What is the magnitude and direction for this vector?

c. Two people are pulling on a stubborn mule. A top-down view reveals that person A is pulling the mule at an angle of 60° with a force of 120 N, person B is pulling the mule at an angle of 165° with a force of 80.0 N, and the mule is pulling with an unknown force at an angle of 270°. Assuming all of the forces cancel each other out, what is the magnitude of the mule's force?

d. A student stands at the edge of a cliff and throws a stone horizontally over the edge with a speed of 18.0 m/s. The cliff is 50.0 m above the flat, horizontal beach. (a) How long after being released does it take for the stone to hit the ground? (b) With what speed and angle does the stone impact the ground with upon landing? Neglect air resistance.

e. One of the fastest recorded pitches in major-league baseball, thrown by Billy Wagner in 2003, was clocked at 101.0 mi/h. If a pitch were thrown horizontally with this velocity, how far would the ball fall vertically by the time it reached home plate, 60.5 feet away? Neglect air resistance.

f. A projectile is launched with an initial speed of 60.0 m/s at an angle of 30.0° above the horizontal. The projectile lands on a hillside 4.00 s later. Neglect air friction. (a) What is the projectile's velocity at the highest point of its trajectory? (b) What is the displacement (straight-line distance) between the projectile and its launching point?

Answer Key

Conversion

a. 1 mile = 1609 meters
1 hour = 3600 seconds

$$\frac{28\,meters}{1\,second} * \frac{1\,mile}{1609\,meters} * \frac{3600\,seconds}{1\,hour} = 62.6\,mph$$

b. 1 mile = 1609 meters
1 hour = 3600 seconds

$$\frac{3.00 * 10^8\,m}{1\,s} * \frac{1\,mile}{1609\,m} * \frac{3600\,s}{1\,hr} = 6.71 * 10^8\,mph$$

c. 25 acre-ft = 25 acre * 1 foot
1 acre = 43560 ft^2
1 acre-ft = 43560 ft^3
1 meter = 3.281 ft

$$25\,acre\,ft * \frac{43560ft^2}{1\,acre} = 1{,}089{,}000ft^3$$

$$1{,}089{,}000\,ft^3 * \left(\frac{1\,m}{3.281\,ft}\right)^3 = 30832.53m^3$$

d. $1m^3$ = 1000 Liters
1 gallon = 3.786L
1 gallon = 4 qts

$$1\,qt * \frac{1\,gal}{4\,qt} * \frac{3.786\,L}{1\,gal} * \frac{1m^3}{1000L} * \left(\frac{100cm}{1m}\right)^3 = 946.5\,cm^3$$

The length of a side is $\sqrt[3]{946.5} = 9.818\,cm$

e. 1 acre = $43560ft^2$
1 meter = 3.281 ft
Volume = bh/3

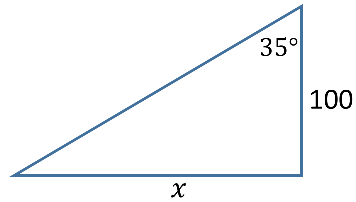
$$13\,acres * \frac{43560ft^2}{1\,acre} * 481\,ft * \frac{1}{3} * \left(\frac{1\,meter}{3.281ft}\right)^3 = 2.57 * 10^6m^3$$

f. Radius = $5.85 * 10^7$m
Mass = $5.68 * 10^{26}$ kg
Density = M/V
Volume = $4/3\pi r^3$
SA = $4\pi r^2$

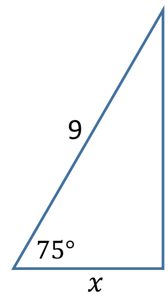
$$\frac{5.68 * 10^{26}kg}{\frac{4}{3}\pi(5.85 * 10^7)^3m^3} * \frac{1000g}{1\,kg} * \left(\frac{1m}{100cm}\right)^3 = .677g/cm^3$$

$$4\pi(5.85 * 10^7\ m)^2 * \left(\frac{3.281ft}{1m}\right)^2 = 4.63 * 10^{17} ft^2$$

Trigonometry



a. $x = 100\tan(35°) = 70.02m$



b. $9.00\cos(75°) = 2.33m$

c. $\sqrt{7^2 + 5^2} = \sqrt{74} = 8.60m$

d. $x = rcos(\theta) = 2.5 \cdot cos(35°) = 2.05$
$y = rsin(\theta) = 2.5 \cdot sin(35°) = 1.43$

e. $\sqrt{8^2 + 1^2} = \sqrt{65} = 8.06$

f. $tan^{-1}(4/3) = 53.13°$
$tan^{-1}(3/5) = 30.96°$
$180° - (53.13° + 30.96°) = 95.91°$

Motion in One Dimension

a. $v_f = 10.97\frac{km}{s}\ or\ 10970\frac{m}{s}$
$v_i = 0$

$\Delta x = 220m$

$$(v_f)^2 = (v_i)^2 + 2a\Delta x$$

$$\left(10970\frac{m}{s}\right)^2 = 2a(220m)$$

$$a \approx 274000\frac{m}{s^2}\ or\ 28000\ times\ the\ force\ of\ gravity$$

$$Not\ possible\ for\ humans$$

b. This problem has three parts, acceleration, constant motion, and deacceleration. The problem asks for total time and total distance

Part one:
$a = 2\ m/s^2$
$v_i = 0\ m/s$
$v_f = 20 m/s$

$(v_f)^2 = (v_i)^2 + 2a\Delta x$
$(20)^2 = (0)^2 + 2(2)\Delta x$
$\Delta x = 100\ m$

$v_f = v_i + at$
$20 = 0 + 2t$
t = 10 seconds

Part Two:
$a = 0$
$\Delta v = 0$
$t = 20\ seconds$
$\Delta x = v * t = 20 * 20 = 400m$

Part Three:
$v_i = 20\ m/s$
$v_f = 0\ m/s$
$t = 5\ sec$

$v_f = v_i + at$
$0 = 20 + 5a$
$a = -4\ m/s^2$

$\Delta x = v_i t + \frac{1}{2}at^2$
$\Delta x = 20(5) + \frac{1}{2}(-4)(5)^2 = 50m$

Answers:
$10sec + 20sec + 5sec = 35sec$
$100m + 400m + 50m = 550m$

c. $\Delta x = 400m$
$v_i = \frac{82.4km}{hr} \cdot \frac{1hr}{3600s} \cdot \frac{1000m}{1km} = 22.89\frac{m}{s}$
$v_f = \frac{16.4km}{hr} \cdot \frac{1hr}{3600s} \cdot \frac{1000m}{1km} = 4.56\frac{m}{s}$
$\Delta x = \frac{1}{2}(v_f + v_i)t$
$400 = \frac{1}{2}\left(22.89\frac{m}{s} + 4.56\frac{m}{s}\right)t$
$t = 29.13\ s$

d. $\Delta x = v_i t + \frac{1}{2}at^2$
$0 = v_i(2) + \frac{1}{2}(-9.8)(2)^2$
$v_i = 9.8\frac{m}{s}$
$\Delta x = (9.8)(1) + \frac{1}{2}(-9.8)(1)^2$
$\Delta x = 4.95m$

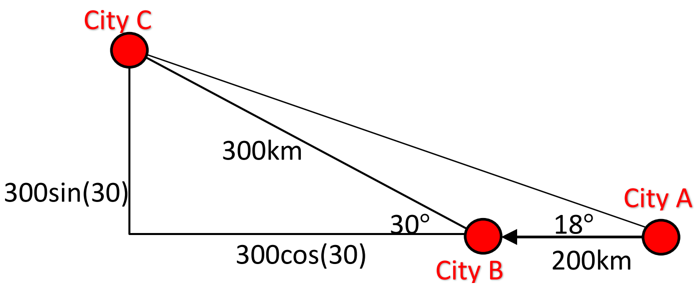
e. Two Parts

a) $t = 2\ seconds$
$a = -9.8\ m/s^2$
$v_f = v_i + at$
$v_f = -1.5 + (-9.8)(2) = -21.1\ m/s$

b) $\Delta x = v_i t + \frac{1}{2}at^2$ (at releasing point, $v_i = 0$)
$\Delta x = (-1.5\frac{m}{s})(2s) + \frac{1}{2}(-9.8)(2)^2$
$\Delta x = -22.6$
$Since\ the\ helicopter\ has\ descended\ 1.50\frac{m}{s} for\ 2\ seconds, 22.6 - (2 * 1.5) = 19.6m$

Motion in Two Dimension

a.



$-200\boldsymbol{i} + 0\boldsymbol{j}$

$-300\cos(30)\,\boldsymbol{i} + 300\sin(30)\,\boldsymbol{j}$

Added together equals -460***i*** + 150***j***

$\sqrt{460^2 + 150^2} = 484km$

$\tan^{-1}\left(\frac{150}{460}\right) = 18°$

b. $-400\cos(40)\,\boldsymbol{i} + 400\sin(40)\,\boldsymbol{j}$
$-100\sin(15)\,\boldsymbol{i} - 100\cos(15)\,\boldsymbol{j}$
$900\cos(30)\,\boldsymbol{i} + 900\sin(30)\,\boldsymbol{j}$
$0\boldsymbol{i} - 100\boldsymbol{j}$
$= 447\boldsymbol{i} + 510\boldsymbol{j}$

$\sqrt{447^2 + 510^2} = 678.17m$
$\theta = \tan^{-1}\left(\frac{510}{447}\right) = 48.77°$
678m at 48.77° above east

c. $120\sin(60°) + 80\sin(165°) - x = 0$
Force of mule = x = 125N

d. $v_{i,x} = 18.0\ m/s$
$v_{i,y} = 0$
$a_y = -9.8\ m/s^2$
$\Delta y = -50$

$v_{f,x} = v_{i,x} + a_x t$ (no horizontal acceleration) → $v_{f,x} = v_{i,x}$
$v_{f,y} = v_{i,y} + a_y t$ → $v_{f,y} = \left(-9.8\frac{m}{s^2}\right)t$
At the time of impact,

$$\Delta x = v_{i,x}t + \frac{1}{2}a_x t^2 = v_{i,x}t$$
$$\Delta y = v_{i,y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}\left(-9.8\frac{m}{s^2}\right)t^2 = -50$$
$$t = 3.19\ s$$

Since $v_{f,x} = v_{i,x} = 18.0\ m/s$ and
$v_{f,y} = \left(-9.8\frac{m}{s^2}\right)(3.19s) = -31.26\frac{m}{s}$, its magnitude

$$v_f = \sqrt{\left(18.0\frac{m}{s}\right)^2 + \left(-31.26\frac{m}{s}\right)^2} = 36.07\frac{m}{s}$$

$\tan^{-1}\left(\frac{-31.26}{18.0}\right) = -60.07°$

e. $\left(\frac{101\ miles}{1\ hour}\right)\left(\frac{1609m}{1mile}\right)\left(\frac{1hr}{3600sec}\right) = 45.14\ m/s$

$(60.5\ ft)\left(\frac{1m}{3.281ft}\right)$ = 18.44m

$t = \frac{18.44m}{45.14m/s} = 0.41\ seconds$

$\Delta y = v_i t + \frac{1}{2}at^2$

$\Delta y = 0 + \frac{1}{2}(-9.8)(0.41)^2 = -0.82\ m$

The ball fell $0.82m$.

f. $v_{i,x} = \left(60\frac{m}{s}\right)\cos(30°)$

$a_x = 0$

$v_{i,y} = \left(60\frac{m}{s}\right)\sin(30°)$

$a_y = -9.8\frac{m}{s^2}$

$t = 4.00s$

$v_{top,y} = 0$

$v_{f,x} = v_{i,x} + a_x t$ (no horizontal acceleration) $v_{f,x} = v_{i,x} = \left(60\frac{m}{s}\right)\cos(30°) = 51.96\ m/s$

$\Delta x = \left(60\frac{m}{s}\right)\cos(30°)\ (4s) = 207.85\ m$

$\Delta y = \left(60\frac{m}{s}\right)\sin(30°)\ (4s) + \frac{1}{2}\left(-9.8\frac{m}{s^2}\right)(4s)^2 = 41.60\ m$

$\sqrt{(207.85)^2 + (41.60m)^2} = 211.97m$

$\tan^{-1}\left(\frac{41.60}{207.85}\right) = 11.32°$