

# Statistics Workshop (STA-2023)

## Confidence Intervals and Hypothesis Testing

### Estimating a Population Proportion

A point estimate is the value of a statistic that estimates the value of a parameter. The point estimate for a population proportion is  $\hat{p} = \frac{x}{n}$ , where  $x$  is the number of individuals in the sample with a specified characteristic and  $n$  is the sample size.

A confidence interval for an unknown parameter consists of an interval of numbers based on a point estimate. The level of confidence represents the expected proportion of intervals that will contain the parameter if a large number of different samples is obtained. In other words, confidence level refers to our confidence in the method.

The level of confidence is denoted as  $(1 - \alpha) * 100\%$ , and the confidence interval estimates for the population proportion are of the form: point estimate  $\pm$  margin of error (E) or

$$\hat{p} \pm E = \hat{p} \pm Z_{\alpha/2} * \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

The margin of error (E) is  $Z_{\alpha/2} * \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$ , where  $Z_{\alpha/2}$  is the critical values of the distribution.

To construct a  $(1 - \alpha) * 100\%$  confidence interval for a population proportion: suppose that a simple random sample of size  $n$  is taken from a population or the data are the result of a randomized experiment. A  $(1 - \alpha) * 100\%$  confidence interval for  $p$  is given by the following quantities:

$$\text{Lower bound: } \hat{p} - Z_{\alpha/2} * \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \quad \text{and} \quad \text{Upper bound: } \hat{p} + Z_{\alpha/2} * \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

(It must be the case that  $n\hat{p}(1 - \hat{p}) \geq 10$  and  $n \leq 0.05N$  to construct this interval.)

To find  $Z_{\alpha/2}$ , first find  $\alpha$ :  $1 - \text{confidence level}$ . Then find  $\alpha/2$ . This is the area under the normal distribution curve, or the probability of getting a certain z-score. You can then use technology or the standard normal table to find the z-score, which in this case would be the critical value  $Z_{\alpha/2}$ .

The most common confidence levels and critical values are:

Confidence level	Area ( $\alpha/2$ )	Critical value ( $Z_{\alpha/2}$ )
90%	0.05	1.645
95%	0.025	1.96
99%	0.005	2.575

Interpretation of a confidence interval:

*A  $(1 - \alpha) * 100\%$  confidence interval indicates that  $(1 - \alpha) * 100\%$  of all simple random samples of size  $n$  from the population whose parameter is unknown will result in an interval that contains the parameter.*

In other words, the interpretation of a confidence interval is this:

*We are (insert level of confidence) confident that the population proportion is between (lower bound) and (upper bound).*

This is an abbreviated way of saying that the method is correct  $(1 - \alpha) \cdot 100\%$  of the time.

The higher, or increasing, the level of confidence leads to a wider interval (increases  $E$ ), whereas increasing the sample size  $n$  will lead to a small interval (decreases  $E$ ) since the larger sample sizes produce more precise estimates.

The sample size required to obtain a  $(1 - \alpha) * 100\%$  confidence interval for  $p$  with the margin of error  $E$  is given by  $n = \hat{p}(1 - \hat{p}) \left( \frac{Z_{\alpha/2}}{E} \right)^2$ , which is rounded up to the **next integer**, where  $\hat{p}$  is a prior estimate of  $p$ .

If a prior estimate,  $\hat{p}$ , is unavailable, we let  $\hat{p} = 0.5$  and use the sample size required is  $n = 0.25 \left( \frac{Z_{\alpha/2}}{E} \right)^2$  rounded up to the next integer. Margin of error should always be expressed as a decimal when using these formulas.

### Example Questions

- Construct a 95% confidence interval of the population proportion using the given information.  $x = 55, n = 100$ 
  - The lower bound is \_\_\_\_\_.
  - The upper bound is \_\_\_\_\_.
- A researcher wishes to estimate the proportion of adults who have high-speed internet access. What size sample should be obtained if she wishes the estimate to be within 0.04 with 99% confidence if
  - She uses a previous estimate of 0.56?
  - She does not use any prior estimate?
- In a survey of 2045 adults in a certain country conducted during a period of economic uncertainty, 58% thought that wages paid to workers in industry were too low. The margin of error was 5 percentage points with 95% confidence. Below are interpretations of the survey results.

- a. In 95% of samples of adults in the country during the period of economic uncertainty, the proportion who believed wages paid to workers in industry were too low is between 0.53 and 0.63. Is this interpretation reasonable?
- b. We are 95% confident that the interval from 0.53 to 0.63 contains the true proportion of adults in the country during the period of economic uncertainty who believe wages paid to workers in industry were too low. Is this interpretation reasonable?

### Estimating a Population Mean

A confidence interval for the population mean is of the form point estimate  $\pm$  margin of error, however now we use mean and standard deviation instead of the population proportion.

Suppose that a simple random sample of size  $n$  is taken from a population. If the population from which the sample is drawn follows a normal distribution, the distribution of  $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$  follows student's t-distribution with  $n - 1$  degrees of freedom, where  $\bar{x}$  is the sample mean and  $s$  is the sample standard deviation.

The properties of the t-distribution:

- 1) The t-distribution is different for different degrees of freedom.
- 2) The t-distribution is centered at 0 and symmetric about 0.
- 3) The area under the curve is 1. The area under the curve to the right of 0 equals the area under the curve to the left of 0, which equals  $\frac{1}{2}$ .
- 4) As  $t$  increases or decreases without bound, the graph approaches but never equals 0.
- 5) The area in the tails (left or right) of the t-distribution is a little greater than the area in the tails of the standard normal distribution because we are using  $s$  as an estimate of  $\sigma$ , thereby introducing further variability into the t-statistic.
- 6) As the sample size increases, the density curve of  $t$  gets closer to the standard normal density curve. This result occurs because, as the sample size increases, the value of  $s$  gets closer to the value of  $\sigma$ , by the Law of Large Numbers.

To construct a  $(1 - \alpha) * 100\%$  confidence interval for  $\mu$ , provided:

- Sample data come from a simple random sample or randomized experiment
- Sample size is small relative to the population size ( $n \leq 0.05N$ ), and
- The data comes from a population that is normally distributed, or the sample size is large.

$$\text{Lower bound: } \bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \quad \text{and} \quad \text{Upper bound: } \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

where  $t_{\alpha/2}$  is the critical value with  $n - 1$  degrees of freedom. Because this confidence interval uses the t-distribution, it is often referred to as a t-interval. In this confidence interval, the margin of error  $E$  is  $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

If the requirements for computing a t-interval are not met, we can

- Increase the sample size beyond 30 observations
- Use a non-parametric procedure, which are inferential procedures that make no assumptions about the underlying distribution of the data (less efficient due to the large sample size)

The sample size required to estimate the population mean,  $\mu$ , with a level of confidence  $(1 - \alpha) * 100\%$  within a specified margin of error,  $E$ , is given by  $n = \left( \frac{Z_{\alpha/2} * s}{E} \right)^2$  where  $n$  is rounded up to the nearest whole number.

When determining sample size for the interval of the population mean we use  $Z_{\alpha/2}$  because  $t_{\alpha/2}$  requires knowing sample size for degrees of freedom.

#### Example questions

- Determine the t-value in each of the cases.
  - Find the t-value such that the area in the right tail is 0.02 with 14 degrees of freedom.
  - Find the t-value such that the area in the right tail is 0.15 with 19 degrees of freedom
  - Find the critical t-value that corresponds to 99% confidence. Assume 10 degrees freedom.
- A survey was conducted that asked 1007 people how many books they had read in the past year. Results indicated that  $\bar{x} = 12.6$  and  $s = 14.8$  books. Construct a 90% confidence interval for the mean number of book people read. Interpret the interval.
  - There is a 90% (confidence/probability) that the population mean number of books read is between \_\_\_\_\_ and \_\_\_\_\_.
- The data shown below represent the repair cost for a low-impact collision in a simple random sample of mini- and micro-vehicles.

\$3146	\$2058	\$1763	\$1037	\$3351
\$742	\$2624	\$1308	\$665	\$771

- Construct and interpret a 95% confidence interval for the population mean cost of repair.

(\_\_\_\_\_, \_\_\_\_\_)

- b. Suppose you obtain a simple random sample of size  $n = 10$  of a specific type of mini-vehicle that was in a low impact collision and determine the cost of repair. Do you think a 95% confidence interval would be wider or narrower?
4. People were polled on how many books they read the previous year. Initial survey results indicate that  $s = 13.7$  books.
- a. How many subjects are needed to estimate the mean number of books read the previous year within six books with 90% confidence?  
This 90% confidence level requires subjects \_\_\_\_\_
- b. How many subjects are needed to estimate the mean number of books read the previous year within three books with 90% confidence?  
This 90% confidence level requires subjects \_\_\_\_\_
- c. What effect does doubling the required accuracy have on the sample size?  
It \_\_\_\_\_ the sample size.
- d. How many subjects are needed to estimate the mean number of books read the previous year within six books with 99% confidence? How does this effect the sample size?  
Increasing the level of confidence (increases/decreases) the sample size required. For a fixed margin of error, greater confidence can be achieved with a (larger/smaller) sample size.

### The Language of Hypothesis Testing

A hypothesis is a proposition assumed as a premise in an argument. In Statistics, a hypothesis is a statement regarding a characteristic of one or more populations.

Hypothesis testing is a procedure, based on a sample evidence and probability, used to test statements regarding a characteristic of one or more populations.

Steps in hypothesis testing:

1. Make a statement regarding the nature of the population.
2. Collect evidence (sample data) to test the statement.
3. Analyze the data to assess the plausibility of the statement.

The null hypothesis, denoted as  $H_0$  (read "H-naught"), is a statement to be tested. The null hypothesis is a statement of no change, no effect, or no difference and is assumed true until evidence indicates otherwise.

The alternative hypothesis, denoted  $H_1$  (read "H-one"), is a statement that we are trying to find evidence to support.

There are three ways to set up the null and alternative hypotheses:

- 1) Equal hypothesis versus not equal hypothesis (two-tailed test)
  - a.  $H_0$ : parameter = some value
  - b.  $H_1$ : parameter  $\neq$  some value
- 2) Equal hypothesis versus a less than hypothesis (left-tailed test)
  - a.  $H_0$ : parameter = somevalue
  - b.  $H_1$ : parameter  $<$  some value (arrow pointing left)
- 3) Equal hypothesis versus a greater than hypothesis (right-tailed test)
  - a.  $H_0$ : parameter = some value
  - b.  $H_1$ : parameter  $>$  some value (arrow pointing right)

When structuring the null and alternative hypotheses:

- Identify the parameter being tested
- Determine the status quo value of this parameter. This gives the null hypothesis.
- Determine the statement that reflects what we are trying to gather evidence for. This gives the alternative hypothesis.

There are four outcomes when doing hypothesis testing:

- 1) Reject the null hypothesis when the alternative hypothesis is true. (correct)
- 2) Do not reject the null hypothesis when the null hypothesis is true. (correct)
- 3) Reject the null hypothesis when the null hypothesis is true. (Type I error)
- 4) Do not reject the null hypothesis when the alternative hypothesis is true. (Type II error)

		Reality	
		$H_0$ Is True	$H_1$ Is True
Conclusion	Do Not Reject $H_0$	Correct Conclusion	Type II Error
	Reject $H_0$	Type I Error	Correct Conclusion

The level of significance,  $\alpha$ , is the probability of making a type I error. As the probability of a type I error increases, the probability of a type II error ( $\beta$ ) decreases, and vice versa.

The last step in any hypothesis test is to state the conclusion to the hypothesis test, which should be in the form:

*There (is/is not) sufficient evidence to conclude that (insert the statement in the alternative hypothesis)*

### Example Questions

1. Three years ago, the mean price of an existing single-family home was \$250,499. A real estate broker believes that existing home prices in her neighborhood are higher.

- a. Determine the null and alternative hypotheses.
  - b. Explain what it would mean to make a Type I error.
  - c. Explain what it would mean to make a Type II error.
2. If the consequences of making a Type I error are severe, would you choose the level of significance,  $\alpha$ , to equal 0.01, 0.05, or 0.10?

### Hypothesis Tests for a Population Proportion

Recall that the best point estimate of  $p$ , the population proportion is given by  $\hat{p} = \frac{x}{n}$ , where  $x$  is the number of individuals with a specified characteristic and  $n$  is the sample size. Also, the  $\mu_{\hat{p}} = p$  and  $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$  provided:

- The sample is a simple random sample
- $np(1 - p) \geq 10$
- The sampled values are independent of each other ( $n \leq 0.05N$ )

When observed results are unlikely under the assumption that the null hypothesis is true, we say the result is statistically significant and we reject the null hypothesis.

Using the classical approach: if the sample statistic is too many standard deviations from the population parameter stated in the null hypothesis, we reject the null hypothesis

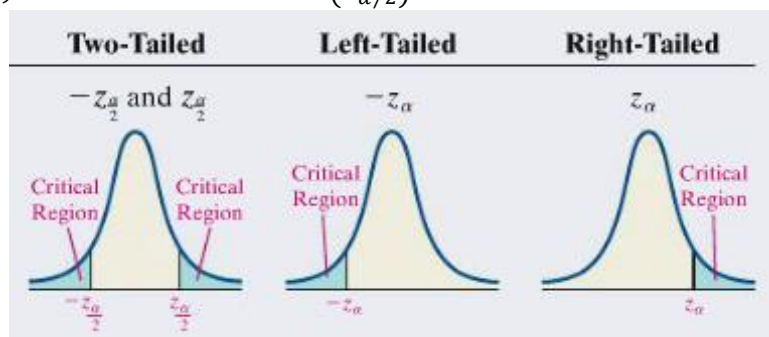
Steps for the classical approach:

- 1) Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways

Two-Tailed	Left-Tailed	Right-Tailed
$H_0: p = p_0$	$H_0: p = p_0$	$H_0: p = p_0$
$H_1: p \neq p_0$	$H_1: p < p_0$	$H_1: p > p_0$
Note: $p_0$ is the assumed value of the population proportion.		

- 2) Select a level of significance,  $\alpha$ , depending on the seriousness of making a Type I error. (usually given)
- 3) Compute the test statistic ( $Z_0$ ) and find the critical value ( $Z_{\alpha/2}$ )

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$



- 4) Compare the critical value and the test statistic

Two-Tailed	Left-Tailed	Right-Tailed
If $z_0 < -z_{\frac{\alpha}{2}}$ or $z_0 > z_{\frac{\alpha}{2}}$ , reject the null hypothesis.	If $z_0 < -z_{\alpha}$ , reject the null hypothesis.	If $z_0 > z_{\alpha}$ , reject the null hypothesis.

- 5) State the conclusion

A p-value is the probability of observing a sample statistic as extreme or more extreme than the one obtained is small under the assumption the statement in the null hypothesis is true, reject the null hypothesis.

Using the p-value approach: if the probability of getting a sample statistic as extreme or more extreme than the one obtained is small under the assumption the statement in the null hypothesis is true, reject the null hypothesis.

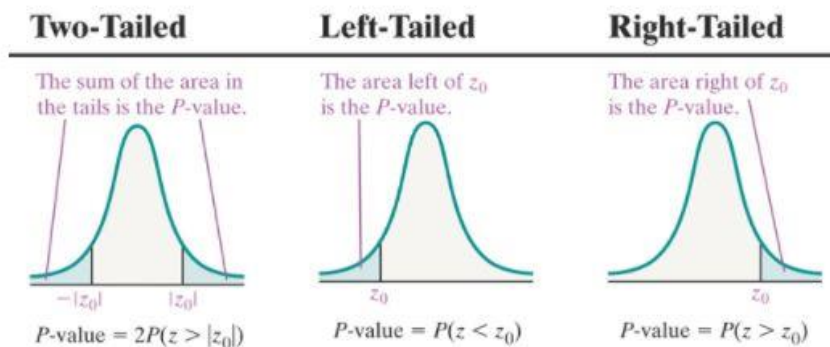
Steps for the p-value approach:

- 1) Determine the null and alternative hypothesis. The hypothesis can be structured in one of three ways.

Two-Tailed	Left-Tailed	Right-Tailed
$H_0: p = p_0$	$H_0: p = p_0$	$H_0: p = p_0$
$H_1: p \neq p_0$	$H_1: p < p_0$	$H_1: p > p_0$

Note:  $p_0$  is the assumed value of the population proportion.

- 2) Select a level off significance,  $\alpha$ , depending on the seriousness of making a Type I error.  
3) Compute the test statistic  $Z_0$  and find the p-value.



- 4) If the p – value  $< \alpha$ , reject the null hypothesis.  
5) State the conclusion.

To do hypothesis testing using a confidence interval:

- When testing  $H_0: p = p_0$  versus  $H_1: p \neq p_0$ , if a  $(1 - \alpha) * 100\%$  confidence interval contains  $p_0$ , DO NOT reject the null hypothesis.



- However, if the confidence interval does not contain  $p_0$ , conclude that  $p \neq p_0$  at the level of significance,  $\alpha$ .

If the requirement  $np(1 - p) \geq 10$  is not satisfied, then calculate  $\hat{p}$  and use binomial distribution, using  $n$  is sample size,  $p$  is probability of success, and  $x$  is the number of successes.

### Examples Questions

1. Suppose a researcher is testing the hypothesis  $H_0: p = 0.3$  versus  $H_1: p \neq 0.3$  and she finds the p-value to be 0.32. Explain what this means. Would she reject the null hypothesis? Why?
2. In a clinical trial, 15 out of 863 patients taking a prescription drug daily complained of flulike symptoms. Suppose that it is known that 1.2% of patients taking competing drugs complain of flulike symptoms. Is there sufficient evidence to conclude that more than 1.2% of these drugs users experience flulike symptoms as a side effect at the  $\alpha = 0.05$  level of significance?
  - a. Because  $np_0(1 - p_0) = \underline{\hspace{2cm}}$  ( $\leq/\geq$ ) 10, the sample size is  $\underline{\hspace{2cm}}$  5% of the population size, and the sample  $\underline{\hspace{2cm}}$  random the requirements for testing the hypothesis  $\underline{\hspace{2cm}}$  satisfied.
  - b.  $H_0: (\mu_0/p_0/\sigma_0) = \underline{\hspace{2cm}}$  versus  $H_1: (\mu_0 / p_0 / \sigma_0) </\neq/> \underline{\hspace{2cm}}$
  - c. Find the test statistic,  $z_0 = \underline{\hspace{2cm}}$  and p-value =  $\underline{\hspace{2cm}}$
  - d. Since (p-value <  $\alpha$ / p-value >  $\alpha$ ) (reject/do not reject) the null hypothesis and conclude that there (is/is not) sufficient evidence that more than 1.2% of users experience flulike symptoms.
3. In a survey, 38% of the respondents stated that they talk to their pets on the telephone. A veterinarian believed this result to be too high, so he randomly selected 185 pet owners and discovered that 62 of them spoke to their pet on the telephone. Does the veterinarian have a right to be skeptical? Use the  $\alpha = 0.05$  level of significance.
  - a. Because  $np_0(1 - p_0) = \underline{\hspace{2cm}}$  ( $\leq/\geq$ ) 10, the sample size is  $\underline{\hspace{2cm}}$  5% of the population size, and the sample  $\underline{\hspace{2cm}}$  random, the requirements for testing the hypothesis  $\underline{\hspace{2cm}}$  satisfied.
  - b.  $H_0: (\mu_0/p_0/\sigma_0) = \underline{\hspace{2cm}}$  versus  $H_1: (\mu_0 / p_0 / \sigma_0) </\neq/> \underline{\hspace{2cm}}$
  - c. Find the test statistic,  $z_0 = \underline{\hspace{2cm}}$  and the p-value =  $\underline{\hspace{2cm}}$

- d. The veterinarian (does not/ does) have a right to be skeptical. There (is/is not) sufficient evidence to conclude that the true proportion of pet owners who talk to their pets on the telephone is (not/less than/greater than) 38%.
4. Several years ago, 45% of parents who had children in grades K-12 were satisfied with the quality of education the students receive. A recent poll asked 1,092 parents who have children in grades K-12 if they were satisfied with the quality of education the students receive. Of the 1,092 surveyed, 470 indicated that they were satisfied. Construct a 99% confidence interval to assess whether this represents evidence that parents' attitudes toward the quality of education have changed.
- $H_0: p_0 = \underline{\hspace{1cm}}$  versus  $H_1: p_0 </\neq/> \underline{\hspace{1cm}}$
  - Use technology to find the 99% confidence interval  
 The lower bound is  $\underline{\hspace{1cm}}$  and the upper bound is  $\underline{\hspace{1cm}}$
  - Since the does/does not contain the proportion stated in the null hypothesis, there is sufficient/insufficient that parents' attitudes toward the quality of education have changed.

### Hypothesis Tests for a Population Mean

To test hypotheses regarding a population mean, use the following steps, provided that

- The sample is obtained using a simple random sampling or from a randomized experiment
- The sample has no outliers and the population from which the sample is drawn is normally distributed, or the sample size,  $n$ , is large ( $n \geq 30$ )
- The sample values are independent of each other

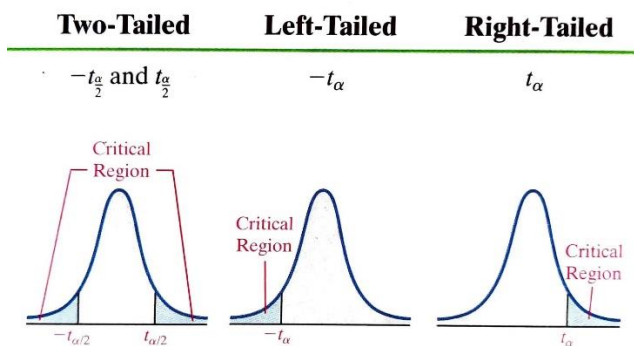
The steps for hypothesis testing using the classical approach:

- Determine the null and alternative hypotheses. The hypotheses can be structured in 3 ways.

Two-Tailed	Left-Tailed	Right-Tailed
$H_0: \mu = \mu_0$	$H_0: \mu = \mu_0$	$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0$	$H_1: \mu < \mu_0$	$H_1: \mu > \mu_0$

Note:  $\mu_0$  is the assumed value of the population mean.

- Select a level of significance,  $\alpha$ , depending on the serious of making a Type I error.
- Compute the test statistic  $t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ , which follows student's t-distribution with  $n - 1$  degrees of freedom.



- 4) Compare the critical value  $t_{\alpha/2}$  to the test-statistic  $t_0$ .

Two-Tailed	Left-Tailed	Right-Tailed
If $t_0 < -t_{\frac{\alpha}{2}}$ or $t_0 > t_{\frac{\alpha}{2}}$ , reject the null hypothesis.	If $t_0 < -t_{\alpha}$ , reject the null hypothesis.	If $t_0 > t_{\alpha}$ , reject the null hypothesis.

- 5) State the conclusion.

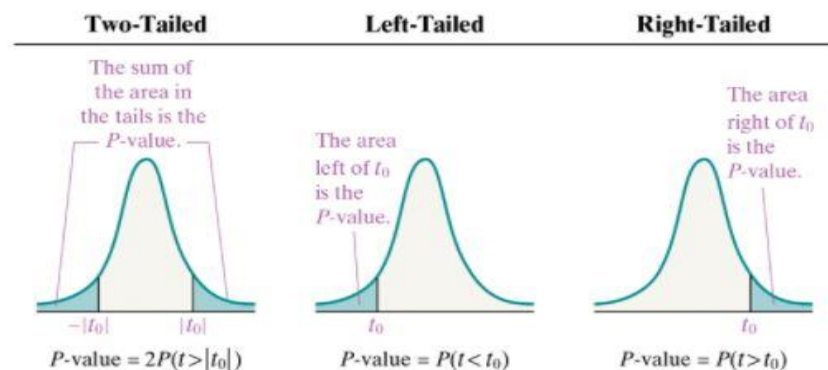
The steps for hypothesis testing using the p-value approach:

- 1) Determine the null and alternative hypothesis. The hypotheses can be structured in one of three ways.

Two-Tailed	Left-Tailed	Right-Tailed
$H_0: \mu = \mu_0$	$H_0: \mu = \mu_0$	$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0$	$H_1: \mu < \mu_0$	$H_1: \mu > \mu_0$

Note:  $\mu_0$  is the assumed value of the population mean.

- 2) Select a level of significance,  $\alpha$ , depending on the seriousness of making a type I error.  
3) Compute the test statistic (t-distribution, with d.f. n-1) and the p-value.



- 4) If the p-value  $< \alpha$ , reject the null hypothesis  
5) State the conclusion.

Practical significance refers to the idea that, while small differences between the statistic and parameter stated in the numerical are statistically significant, the difference may not be large enough to cause or considered important.

In statistical hypothesis testing, a result has statistical significance when it is very unlikely to have occurred given the null hypothesis.

Large sample sizes can lead to results that one statistically significant, while the difference between the statistic parameter in the null hypothesis is not enough to be considered practically significant.

### Example Questions

1. One year, the mean age of an inmate on death row was 42.7 years. A sociologist wondered whether the mean age of a death-row inmate has changed since then. She randomly selects 32 death-row inmates and finds that their mean age is 41.2, with a standard deviation of 8.4. Construct a 95% confidence interval about the mean age. What does the interval imply?
  - a.  $H_0: (\mu_0 / p_0 / \sigma_0) = \underline{\hspace{2cm}}$  versus  $H_1: (\mu_0 / p_0 / \sigma_0) </\neq/> \underline{\hspace{2cm}}$
  - b. Construct a 95% confidence interval about the mean age.  
 With 95% confidence, the mean age of death row inmates is between            and           .
  - c. Since the mean age from the earlier year (is/is not) contained in the interval, there (is/is not) sufficient evidence to conclude that the mean age had changed.
2. To test  $H_0: \mu = 65$  versus  $H_1: \mu < 65$ , a random sample of size  $n = 28$  is obtained from a population that is known to be normally distributed.
  - a. If  $\bar{x} = 57.8$  and  $s = 13.1$ , compute the test statistic.  
 $t_0 = \underline{\hspace{2cm}}$
  - b. If the researcher decides to test this hypothesis at the  $\alpha = 0.05$  level of significance, determine the critical value(s).  
 $t_\alpha = \underline{\hspace{2cm}}$
  - c. Will the researcher reject the null hypothesis?  
 (Yes/No) because the  $t_0$  (does/does not) fall in the critical region.
3. Several years ago, the mean height of women 20 years of age or older was 64.8 inches. Suppose that a random sample of 50 women who are 20 years of age or older today results in a mean height of 65.5 inches.
  - a. State the appropriate null and alternative hypotheses to assess whether women are taller today.

$H_0: (\mu_0 / p_0 / \sigma_0) = \underline{\hspace{2cm}}$  versus  $H_1: (\mu_0 / p_0 / \sigma_0) </\neq/> \underline{\hspace{2cm}}$

- b. Suppose the P-value for this test is 0.02. Explain what this value represents.

There is a 0.02 probability of obtaining a sample mean of            or taller from a population whose mean height is           .

- c. Write a conclusion for this hypothesis test assuming an  $\alpha = 0.10$  level of significance.

(Reject/Do not reject) the null hypothesis. There (is/is not) sufficient evidence to conclude that the mean height of women 20 years of age or older is greater today.

4. According to a food website, the mean consumption of popcorn annually by Americans is 55 quarts. The marketing division of the food website unleashes an aggressive campaign designed to get Americans to consume even more popcorn.

- a. Determine the null and alternative hypotheses that would be used to test the effectiveness of the marketing campaign.

$H_0: (\mu_0 / p_0 / \sigma_0) = \underline{\hspace{2cm}}$  versus  $H_1: (\mu_0 / p_0 / \sigma_0) </\neq/> \underline{\hspace{2cm}}$

- b. A sample 890 of Americans provides enough evidence to conclude that marketing campaign was effective. Provide a statement that should be put out by the marketing department.

There (is/is not) sufficient evidence to conclude that the mean consumption of popcorn has risen.

- c. Suppose, in fact, the mean annual consumption of popcorn after the marketing campaign is 55 quarts. Has a Type I or Type II error been made by the marketing department? If we tested this hypothesis at the level of significance, what is the probability of committing this error?

The marketing department committed a (Type I/Type II) error because the marketing department (rejected/ did not reject) the (null/alternative) hypothesis when it was true. The probability of making a (Type I/Type II) error is           .

## **Answer Key**

### **Estimating a Population Proportion**

- 1) Lower bound: 0.452, Upper bound: 0.648
- 2) (a) 1022, (b) 1037
- 3) (a) No, suggests that this interval sets the standard for all intervals, which is not true.  
 (b) Yes, this is interpretation is reasonable

### **Estimating a Population Mean**

- 1) (a) 2.26, (b) 1.07, (c) 3.17
- 2) Lower bound: 11.832, Upper bound: 13.368
- 3) (a) Lower Bound: 1022.2, Upper Bound: 2470.8, (b) Narrower because of less variability
- 4) (a) 15, (b) 57, (c) quadruples, (d) 35

### **The Language of Hypothesis Testing**

- 1) (a)  $H_0: \mu = 250499$  versus  $H_1: \mu > 250499$ , (b) rejecting the null hypothesis mean of \$250,499 for the mean price single family-home when it is the true mean, (c) Not rejecting the null hypothesis when the alternative hypothesis was true
- 2) 0.01

### **Hypothesis Tests for a Population Proportion**

- 1) 0.32 is the probability of getting at least extreme as the test statistic, which would be 32 out 100. Since this is not unusual do not reject the null hypothesis.
- 2) (a) 10.23,  $\geq$ , less than, can reasonably be assumed to be random, are, (b)  $H_0: p = 0.012, H_1: p > 0.012$ , (c)  $Z_0 = 1.45$  and p-value = 0.073 (d) p-value  $> \alpha$ , do not reject, is not
- 3) (a) 43.59,  $\geq$ , less than, can reasonably assumed to be, are, (b)  $H_0: p = 0.38, H_1: p < 0.38$ , (c)  $Z_0 = -1.26$  and p-value = 0.104, (d) does not, is not, less than
- 4) (a)  $H_0: p = 0.45$  versus  $H_1: p \neq 0.45$ , (b) Lower bound: 0.392, Upper bound: 0.469, (c) Does, insufficient

### **Hypothesis Tests for a Population Mean**

- 1) (a)  $H_0: \mu = 42.7$  versus  $H_1: \mu \neq 42.7$ , (b) Lower bound: 38.171, Upper bound: 44.229, (c) Is, is not
- 2) (a)  $-2.91$ , (b)  $-1.70$ , (c) Yes, does
- 3) (a)  $H_0: \mu = 64.8$  versus  $H_1: \mu > 64.8$ , (b) 65.5, 64.8, (c) Reject, is
- 4) (a)  $H_0: \mu = 55$  versus  $H_1: \mu > 55$ , (b) is, (c) Type I, rejected, null, Type I,  $\alpha$