

Factoring Polynomials

Factoring a polynomial expressed as the sum of monomials means finding an equivalent expression that is a product.

1. In any factoring problem, the first step is to look for the **greatest common factor (GCF)**.
2. Then determine the number of terms in the polynomial and try factoring in proper methods.
3. If there are four or more terms, try factoring by grouping.
4. Check to see if any factors with more than one term in the factored polynomial can be factored further. If so, factor completely until it cannot be factored further.

Type of Polynomial	Method	Example
Any polynomial	Look for common monomial factors. (Always do this first!)	$6x^2 + 9x = 3x(2x + 3)$
Binomials of degree 2 or higher	Check for a special product:	
	Difference of two squares, $a^2 - b^2 = (a - b)(a + b)$	$x^2 - 16 = (x - 4)(x + 4)$
	Difference of two cubes, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$x^3 - 64 = (x - 4)(x^2 + 4x + 16)$
Trinomials of degree 2	Sum of two cubes, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	$x^3 + 27 = (x + 3)(x^2 - 3x + 9)$
	Check for a perfect square, $(x \pm a)^2 = x^2 \pm 2ax + a^2$	$x^2 + 8x + 16 = (x + 4)^2$ $x^2 - 10x + 25 = (x - 5)^2$
	Factoring $x^2 + Bx + C$	$x^2 - x - 2 = (x - 2)(x + 1)$
Four or more terms	Factoring $Ax^2 + Bx + C$	$6x^2 + x - 1 = (2x + 1)(3x - 1)$
	Grouping	$2x^3 - 3x^2 + 4x - 6$ $= (2x - 3)(x^2 + 2)$

Exercise

Factor completely!

1. $18x^3 + 27x^2$

2. $9x^4 - 18x^3 + 27x^2$

3. $x^2(2x + 5) + 17(2x + 5)$

4. $x^3 + 5x^2 - 2x - 10$

5. $3x^3 - 2x^2 - 6x + 4$

6. $x^3 - x^2 - 5x + 5$

7. $x^2 + 13x + 40$

8. $x^2 - 5x - 14$

9. $6x^2 + 19x - 7$

10. $3x^2 - 13xy + 4y^2$

11. $x^2 - 2x - 15$

12. $2x^2 + 5x - 3$

13. $3x^2 - x - 2$

14. $4x^2 + 16x + 15$

15. $8x^2 + 33x + 4$

16. $15x^2 - 19x + 6$

17. $6x^2 - 7xy - 5y^2$

18. $9x^2 - 121y^2$

19. $x^2 - 81$

20. $81x^4 - 16$

21. $x^2 + 14x + 49$

22. $4x^2 + 4x + 1$

23. $16x^2 - 56x + 49$

24. $x^3 + 1$

25. $8x^3 - 1$

26. $64x^3 + 27$

27. $3x^3 - 30x^2 + 75x$

28. $x^2 - 36a^2 + 20x + 100$

29. $x^3 + 3x^2 - 25x - 75$

30. $2x^2 - 2x - 112$

31. $20y^4 - 45y^2$

32. $9b^2x - 16y - 16x + 9b^2y$

33. $x^2y - 16y + 32 - 2x^2$

34. $12x^2y - 27y - 4x^2 + 9$

35. $2x^3 - 8a^2x + 24x^2 + 72x$

36. $x(x-1)^{-\frac{1}{2}} + (x-1)^{\frac{1}{2}}$

37. $4x^{-\frac{2}{3}} + 8x^{\frac{1}{3}}$

38. $12x^{-\frac{3}{4}} + 6x^{\frac{1}{4}}$

39. $(4x-1)^{\frac{1}{2}} - 3(4x-1)^{\frac{3}{2}}$

40. $10x^2(x+1) - 7x(x+1) - 6(x+1)$

41. $12x^2(x - 1) - 4x(x - 1) - 5(x - 1)$

42. $7x^4 + 34x^2 - 5$

43. $(y + 1)^3 + 1$

44. $x^4 - 5x^2y^2 + 4y^4$

45. $(x - y)^4 - 4(x - y)^2$

46. $2x^2 - 7xy^2 + 3y^4$

47. $3x^2 + 5xy^2 + 2y^4$

48. $6x^4 + 35x^2 - 6$

49. $(x + y)^4 - 100(x + y)^2$

50. $y^7 + y$

Answer Key

1. $9x^2(2x + 3)$
2. $9x^2(x^2 - 2x + 3)$
3. $(2x + 5)(x^2 + 17)$
4. $(x + 5)(x^2 - 2)$
5. $(x^2 - 2)(3x - 2)$
6. $(x - 1)(x^2 - 5)$
7. $(x + 8)(x + 5)$
8. $(x - 7)(x + 2)$
9. $(3x - 1)(2x + 7)$
10. $(3x - y)(x - 4y)$
11. $(x - 5)(x + 3)$
12. $(2x - 1)(x + 3)$
13. $(3x + 2)(x - 1)$
14. $(2x + 3)(2x + 5)$
15. $(8x + 1)(x + 4)$
16. $(5x - 3)(3x - 2)$
17. $(2x + y)(3x - 5y)$
18. $(3x + 11y)(3x - 11y)$
19. $(x + 9)(x - 9)$
20. $(9x^2 + 4)(3x + 2)(3x - 2)$
21. $(x + 7)^2$
22. $(2x + 1)^2$
23. $(4x - 7)^2$
24. $(x + 1)(x^2 - x + 1)$
25. $(2x - 1)(4x^2 + 2x + 1)$
26. $(4x + 3)(16x^2 - 12x + 9)$
27. $3x(x - 5)^2$
28. $(x + 10 + 6a)(x + 10 - 6a)$
29. $(x + 3)(x + 5)(x - 5)$
30. $2(x + 7)(x - 8)$
31. $5y^2(2y + 3)(2y - 3)$
32. $(x + y)(3b + 4)(3b - 4)$
33. $(y - 2)(x + 4)(x - 4)$
34. $(3y - 1)(2x + 3)(2x - 3)$
35. $2x(x + 6 + 2a)(x + 6 - 2a)$
36. $(x - 1)^{-\frac{1}{2}}(2x - 1)$
37. $4x^{-\frac{2}{3}}(1 + 2x)$
38. $6x^{-\frac{3}{4}}(2 + x)$
39. $4(4x - 1)^{\frac{1}{2}}(1 - 3x)$
40. $(x + 1)(2x + 1)(5x - 6)$
41. $(x - 1)(2x + 1)(6x - 5)$
42. $(7x^2 - 1)(x^2 + 5)$
43. $(y + 2)(y^2 + y + 1)$
44. $(x + y)(x - y)(x + 2y)(x - 2y)$
45. $(x - y)^2(x - y + 2)(x - y - 2)$
46. $(2x - y^2)(x - 3y^2)$
47. $(3x + 2y^2)(x + y^2)$
48. $(x^2 + 6)(6x^2 - 1)$
49. $(x + y)^2(x + y + 10)(x + y - 10)$
50. $y(y^2 + 1)(y^4 - y^2 + 1)$