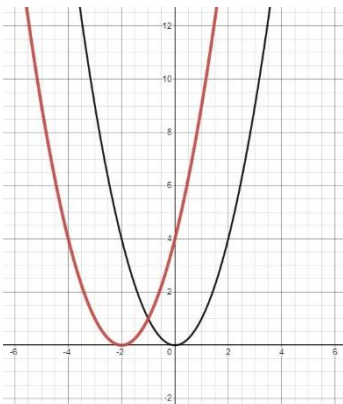
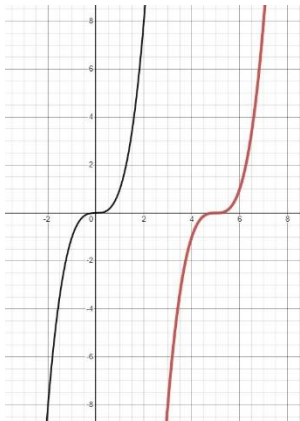
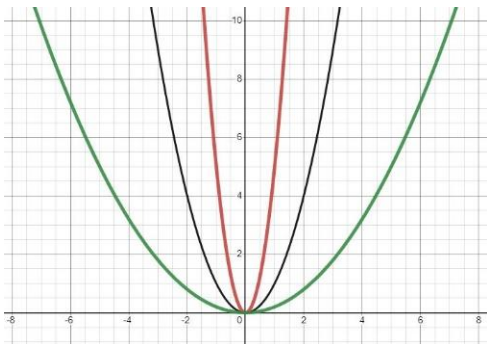
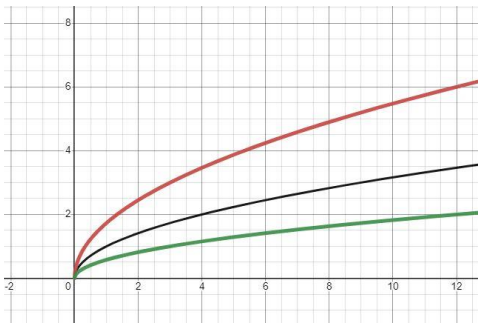


Transformation of the Graphs

Summary of Transformations

Start with the basic graph of $y = f(x)$.

To Graph:	Draw the Graph of f and:	Functional Change to $f(x)$
Horizontal shifts		
$y = f(x + c), \quad c > 0$	Shift the graph of f to the left c units.	Replace x by $x + c$.
$y = f(x - c), \quad c > 0$	Shift the graph of f to the right c units.	Replace x by $x - c$.
Ex) Graph the function, $y = (x + 2)^2$	Ex) Graph the function, $y = (x - 5)^3$	
Start with the basic graph: $y = x^2$	Start with the basic graph: $y = x^3$	
		
Compressing or stretching		
$y = af(x), \quad a > 0$	Multiply each y -coordinate of $y = f(x)$ by a . Stretch the graph of f vertically if $a > 1$. Compress the graph of f vertically if $0 < a < 1$.	Multiply $f(x)$ by a .
$y = f(ax), \quad a > 0$	Multiply each x -coordinate of $y = f(x)$ by $\frac{1}{a}$. Stretch the graph of f horizontally if $0 < a < 1$. Compress the graph of f horizontally if $a > 1$.	Replace x by ax .
Ex) Graph the functions, $y = 5x^2$ and $y = \frac{1}{5}x^2$	Ex) Graph the function, $y = \sqrt{3x}$ and $y = \sqrt{\frac{1}{3}x}$	
Start with the basic graph: $y = x^2$	Start with the basic graph: $y = \sqrt{x}$	
		

Reflection about the x -axis

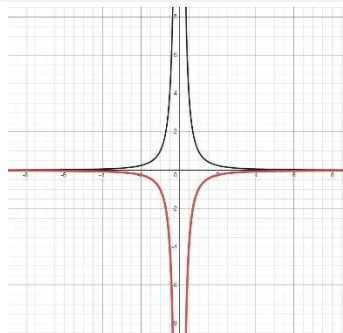
$$y = -f(x)$$

Reflect the graph of f about the x -axis.

Multiply $f(x)$ by -1 .

Ex) Graph the function, $y = -\frac{1}{x^2}$

Start with the basic graph: $y = \frac{1}{x^2}$



Reflection about the y -axis

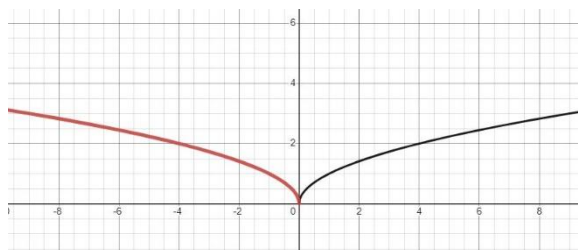
$$y = f(-x)$$

Reflect the graph of f about the y -axis.

Replace x by $-x$.

Ex) Graph the function, $y = \sqrt{-x}$

Start with the basic graph: $y = \sqrt{x}$



Vertical shifts

$y = f(x) + k$, $k > 0$ Raise the graph of f by k units.

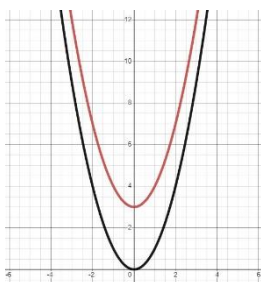
Add k to $f(x)$.

$y = f(x) - k$, $k > 0$ Lower the graph of f by k units.

Subtract k from $f(x)$.

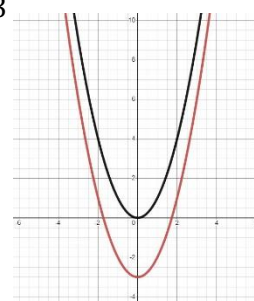
Ex) Graph the function, $y = x^2 + 3$

Start with the basic graph: $y = x^2$



Ex) Graph the function, $y = x^2 - 3$

Start with the basic graph: $y = x^2$



Order of Transformations

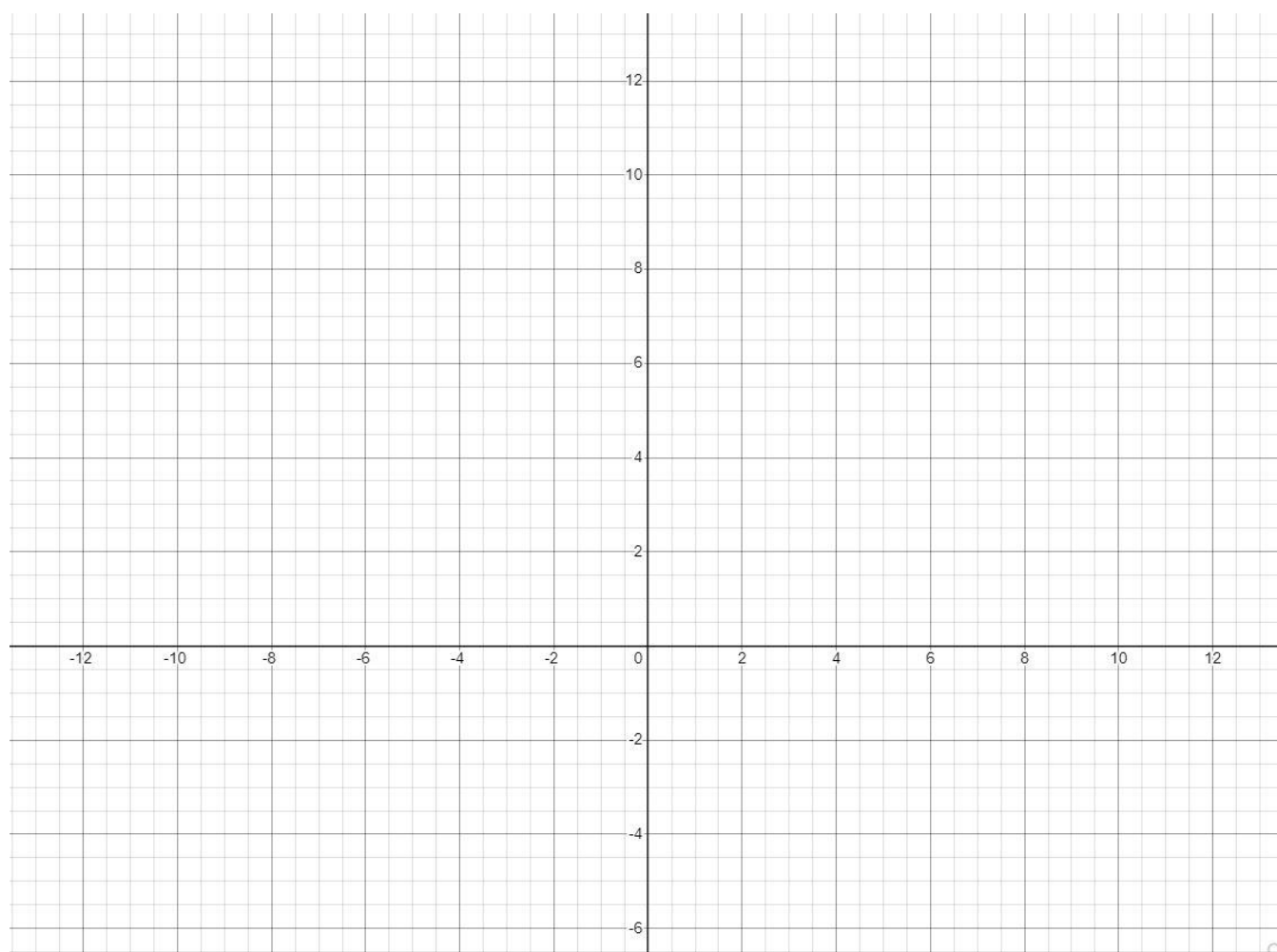
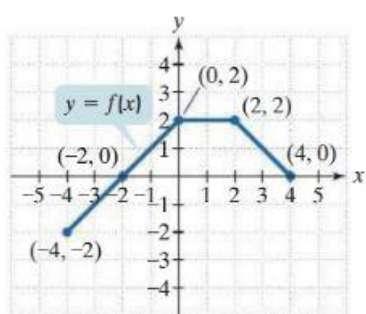
A function involving more than one transformation can be graphed by performing transformations in the following order:

1. Horizontal shifting
2. Stretching or compressing
3. Reflecting
4. Vertical shifting

Exercise

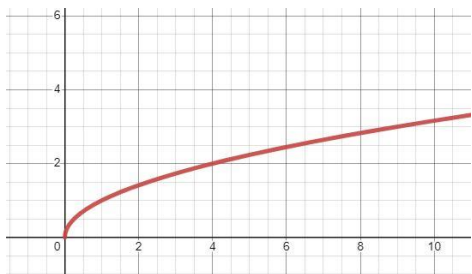
1. Use the given graph of $y = f(x)$ below to obtain the graph of $y = f(-x)$ and $y = 2f(x)$.

From the graph	To find points on the graph, $y = f(-x)$			To find points on the graph, $y = 2f(x)$		
(x, y)	x	$y = f(-x)$	(x, y)	x	$y = 2f(x)$	(x, y)
$(-4, -2)$	-4	$f(-x) = f(4) = 0$	$(-4, 0)$	-4	$2f(x) = 2f(-4) = 2(-2) = -4$	$(-4, -4)$
$(-2, 0)$	-2	$f(-x) = f(2) = 2$		-2		
$(0, 2)$	0			0		
$(2, 2)$	2			2		
$(4, 0)$	4			4		



2. Graph the functions of $f(x) = \sqrt{x-2}$ and $g(x) = \sqrt{x+4}$.

Basic graph: $y = \sqrt{x}$



To obtain the graphs, start with the basic graph $y = \sqrt{x}$.

1) $f(x) = \sqrt{x-2}$

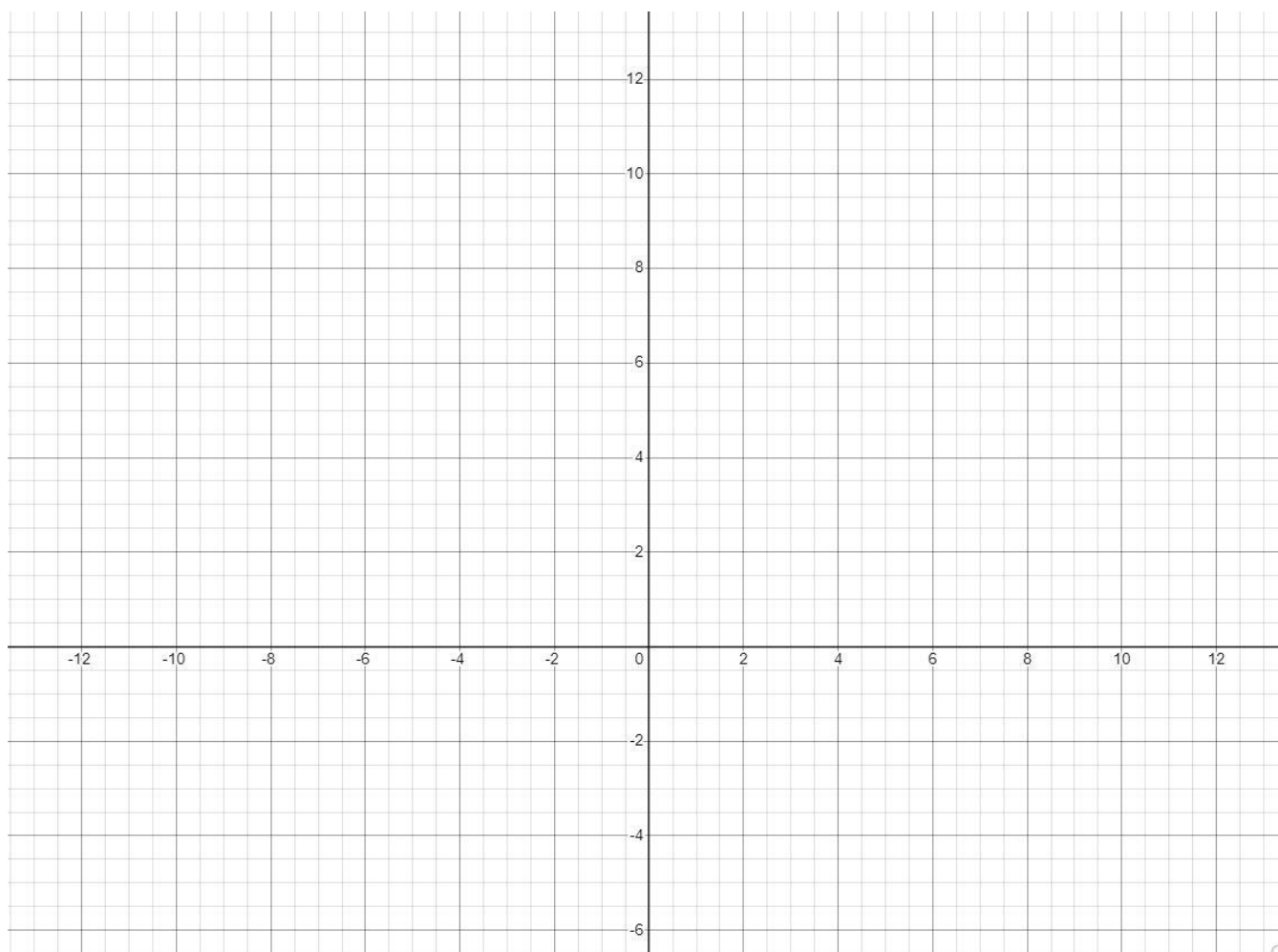
Shift the basic graph horizontally 2 units to the right.

2) $g(x) = \sqrt{x+4}$

Shift the basic graph horizontally 4 units to the left.

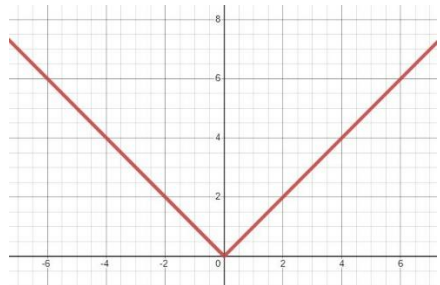
To obtain the specific points on the graphs,

x	$f(x) = \sqrt{x-2}$	$(x, f(x))$	$g(x) = \sqrt{x+4}$	$(x, g(x))$



3. Graph the function of $h(x) = |x + 3| - 2$.

Basic graph: $y = |x|$



To obtain the graph of $h(x)$, start with the basic graph $y = |x|$.

1) Horizontal Shifting: $|x| \rightarrow |x + 3|$

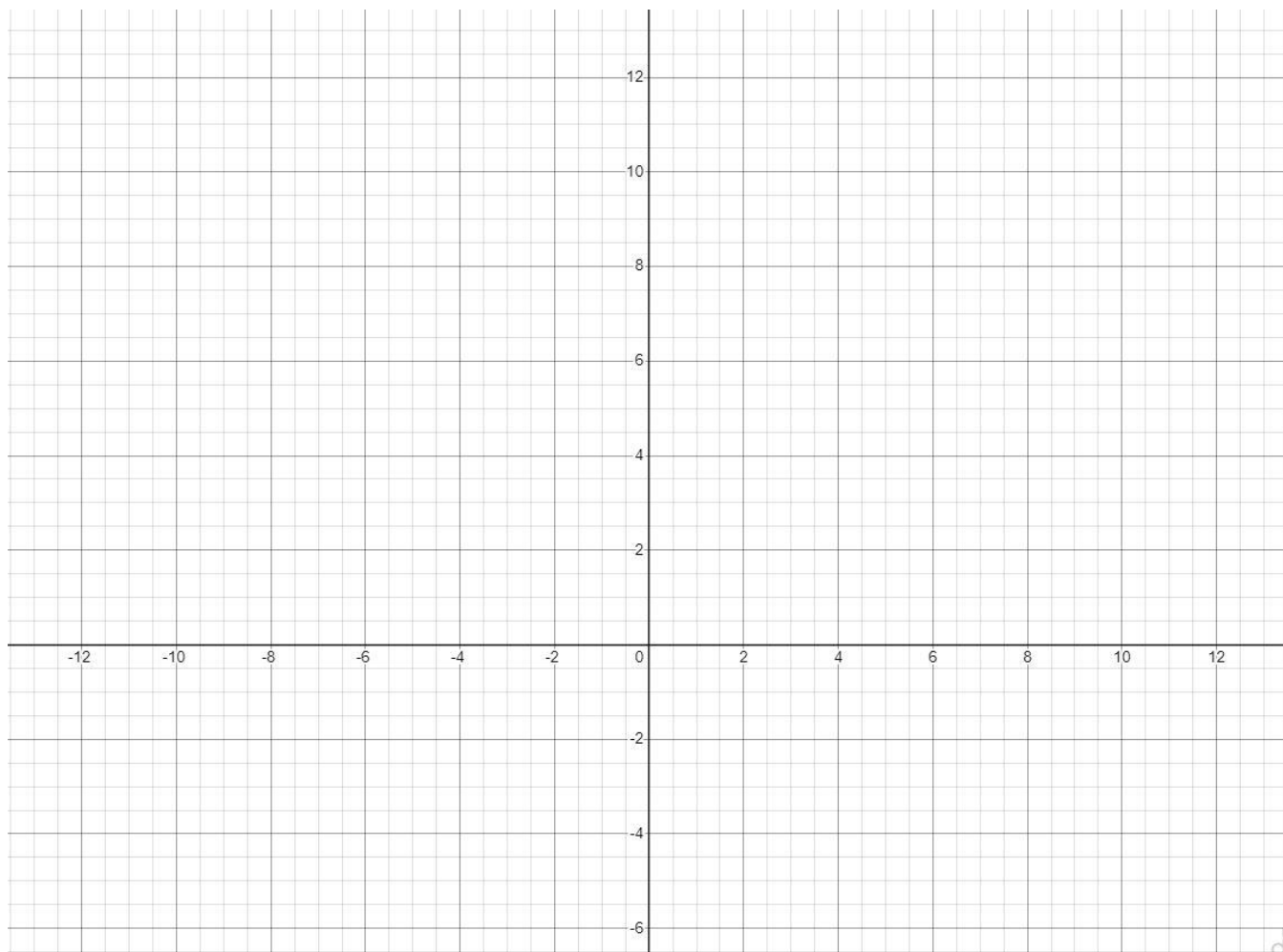
Shift the basic graph horizontally to the left 3 units.

2) Vertical Shifting: $|x + 3| \rightarrow |x + 3| - 2$

Shift the basic graph vertically down 2 units.

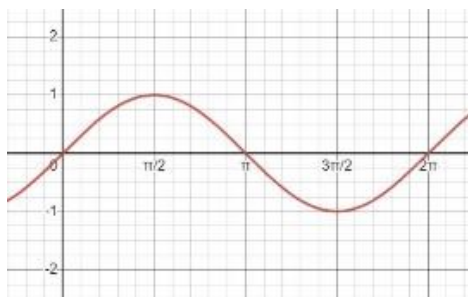
To obtain the specific points on the graph,

x	$h(x) = x + 3 - 2$	$(x, h(x))$



4. Graph the function of $f(x) = 3 \sin x$.

Basic graph: $y = \sin x$



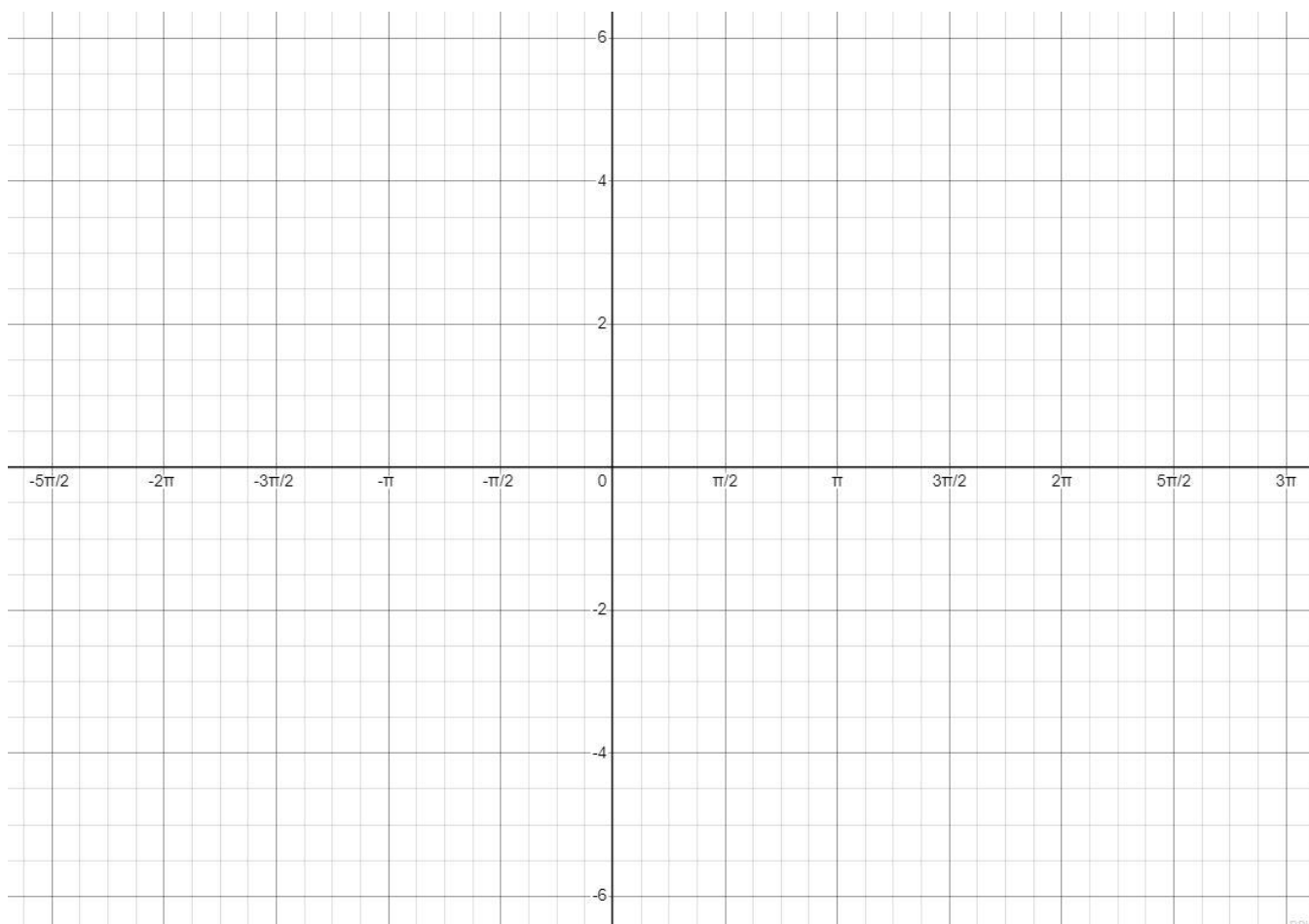
To obtain the graph of $f(x)$, start with the basic graph $y = \sin x$.

Stretching: $\sin x \rightarrow 3 \sin x$

Stretch vertically by a factor of 3

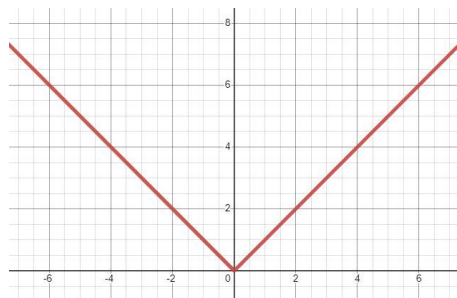
To obtain specific points on the graph,

x	$f(x) = 3 \sin x$	$(x, f(x))$



5. Graph the function of $g(x) = \frac{1}{2}|x|$.

Basic graph: $y = |x|$



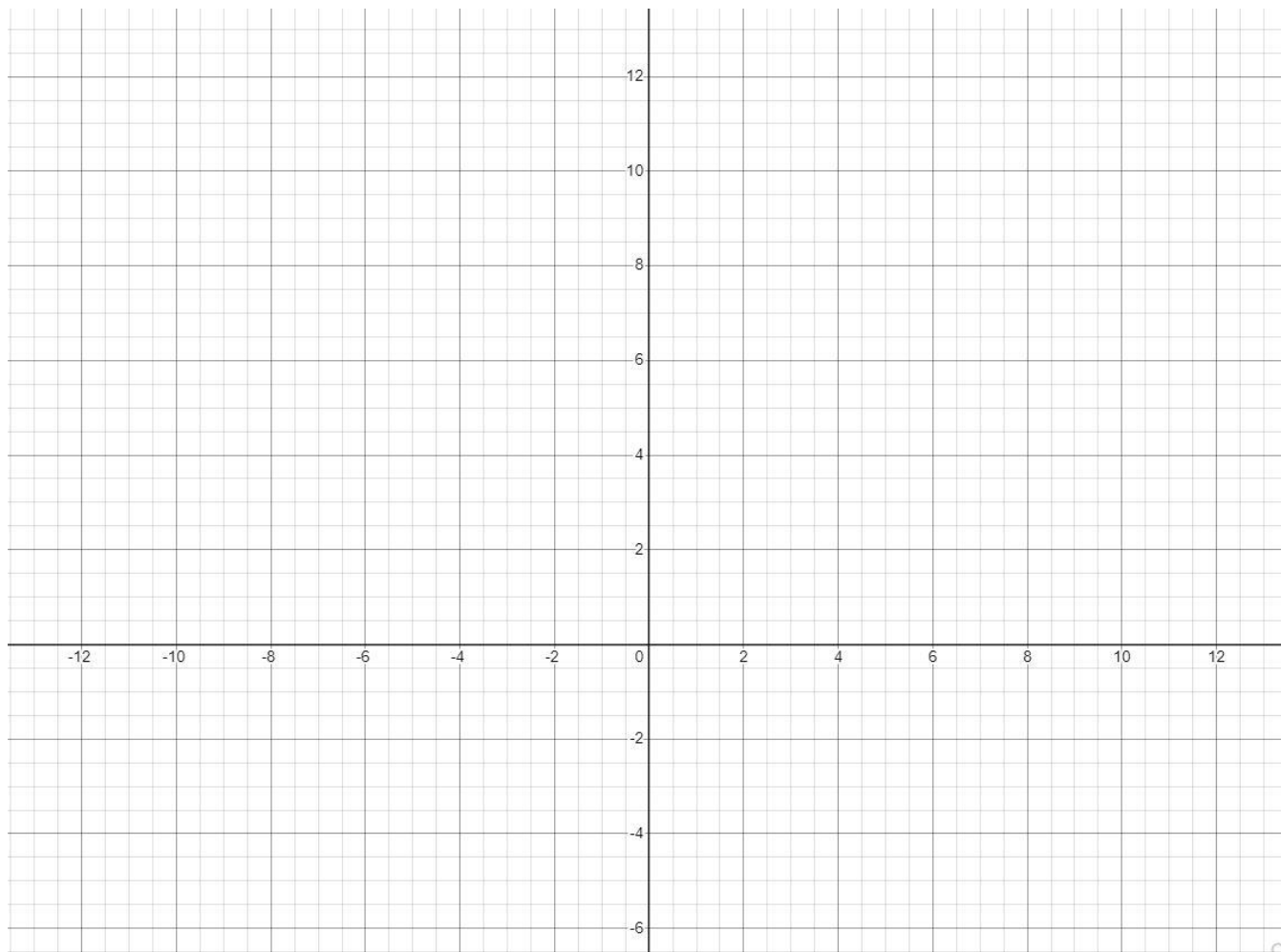
To obtain the graph of $g(x)$, start with the basic graph $y = |x|$.

Compression: $|x| \rightarrow \frac{1}{2}|x|$

Compress vertically by a factor of 1/2

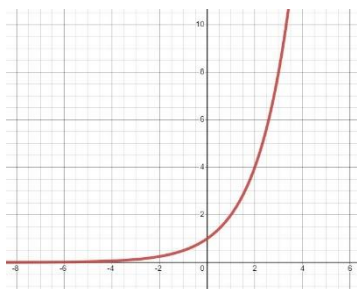
To obtain specific points on the graph,

x	$g(x) = \frac{1}{2} x $	$(x, g(x))$



6. Graph the function of $f(x) = 2^{-x}$ and $g(x) = -2^x$.

Basic graph: $y = 2^x$



To obtain the graph of $f(x)$ and $g(x)$, start with the basic graph $y = 2^x$.

1) $f(x) = 2^{-x}$

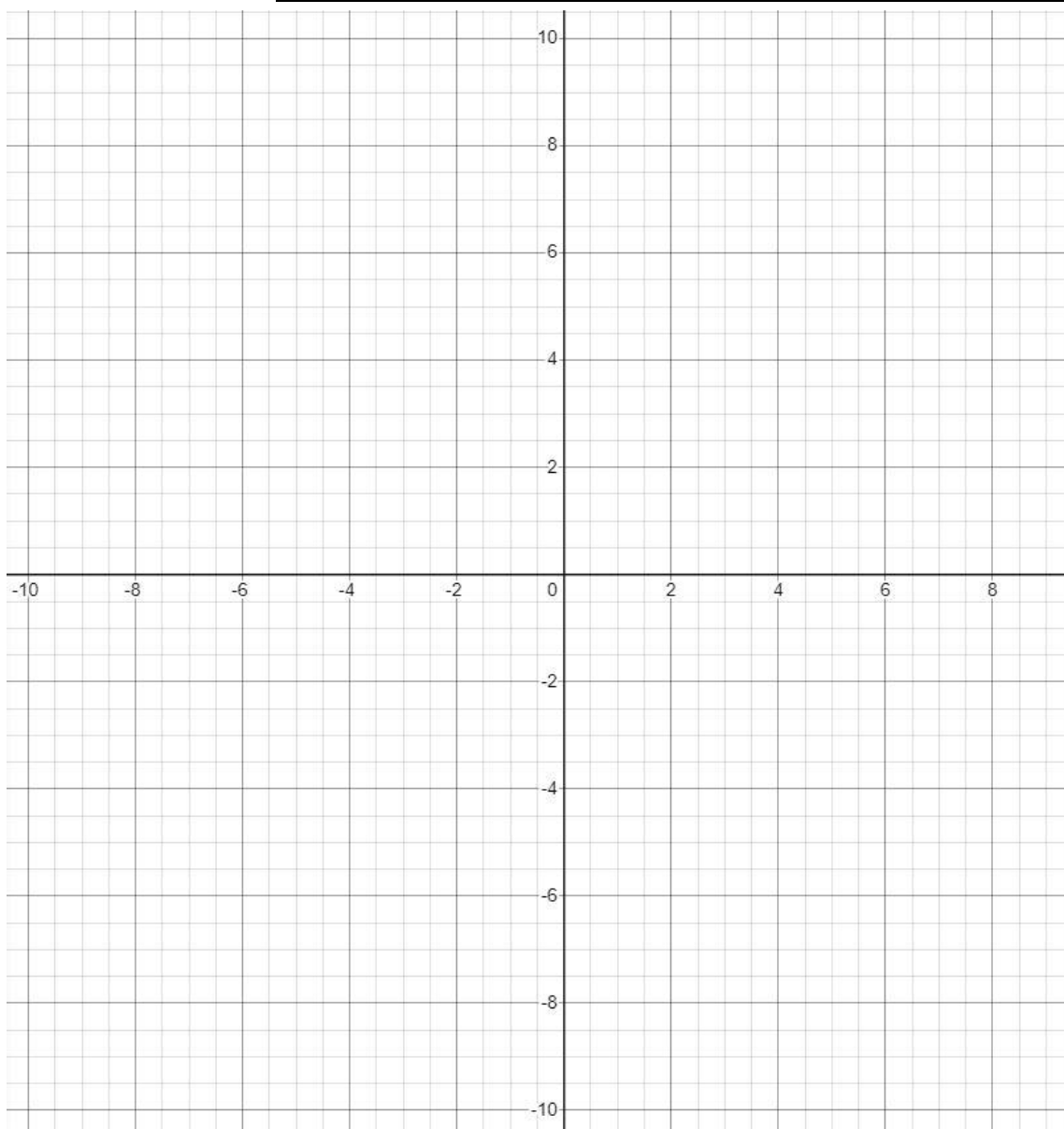
Reflection to the y -axis: $2^x \rightarrow 2^{-x}$

2) $g(x) = -2^x$

Reflection to the x -axis: $2^x \rightarrow -2^x$

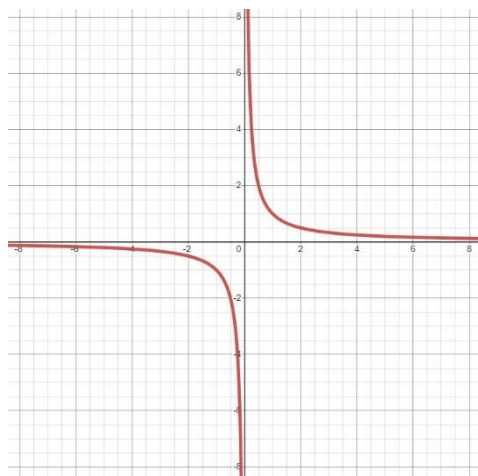
To obtain the specific points on the graphs,

x	$f(x) = 2^{-x}$	$(x, f(x))$	$g(x) = -2^x$	$(x, g(x))$



7. Graph the function $h(x) = \frac{3}{x-2} + 1$.

Basic graph: $y = \frac{1}{x}$

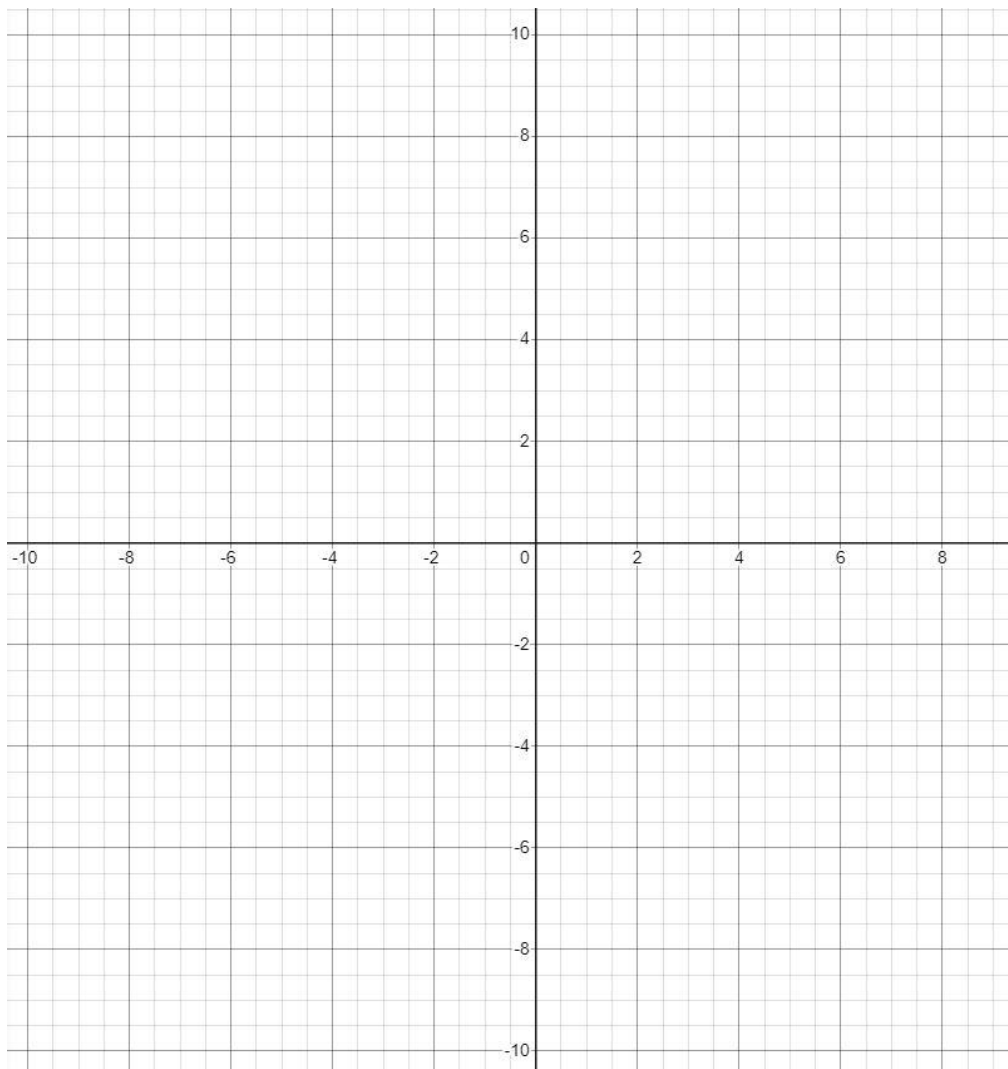


To obtain the graph of $h(x)$, start with the basic graph $y = \frac{1}{x}$.

- 1) Horizontal shifting: $\frac{1}{x} \rightarrow \frac{1}{x-2}$
Shift the basic graph horizontally 2 units to the right.
- 2) Stretching: $\frac{1}{x-2} \rightarrow \frac{3}{x-2}$
Stretch vertically by a factor of 3.
- 3) Vertical shifting: $\frac{3}{x-2} \rightarrow \frac{3}{x-2} + 1$
Shift the graph vertically up 1 unit.

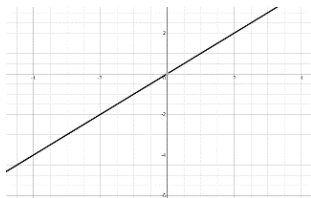
To obtain the specific points on the graph,

x	$h(x) = \frac{3}{x-2} + 1$	$(x, h(x))$

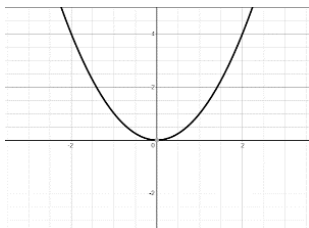


Good Graphs to Know

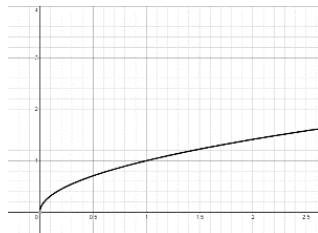
$$f(x) = x$$



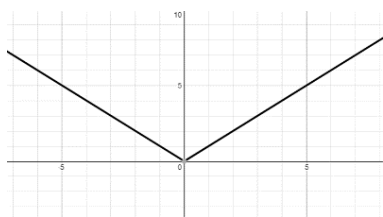
$$f(x) = x^2$$



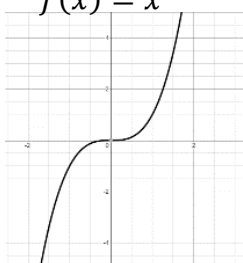
$$f(x) = \sqrt{x}$$



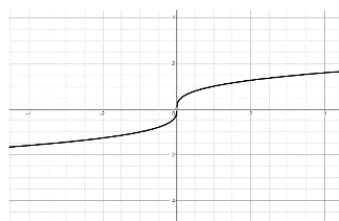
$$f(x) = |x|$$



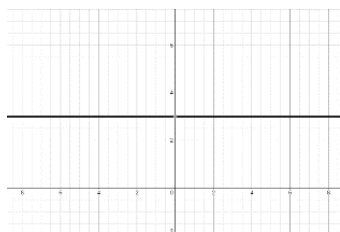
$$f(x) = x^3$$



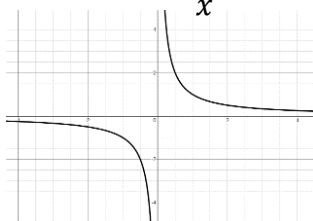
$$f(x) = \sqrt[3]{x}$$



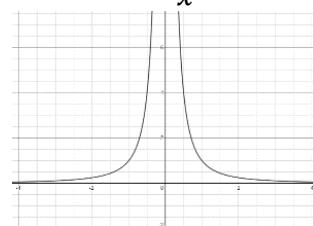
$$f(x) = c, c = 3$$



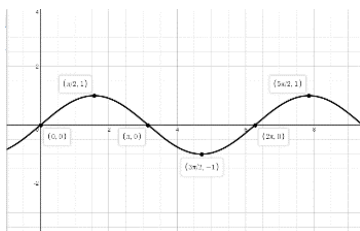
$$f(x) = \frac{1}{x}$$



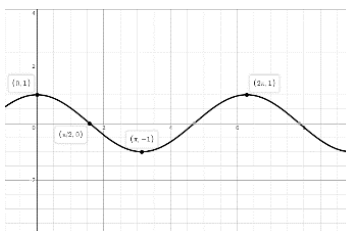
$$f(x) = \frac{1}{x^2}$$



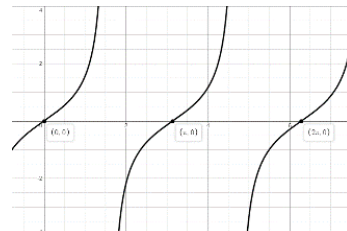
$$f(x) = \sin x$$



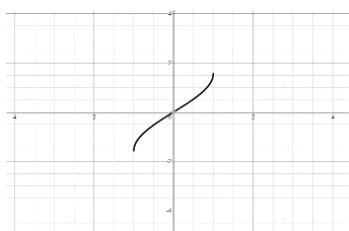
$$f(x) = \cos x$$



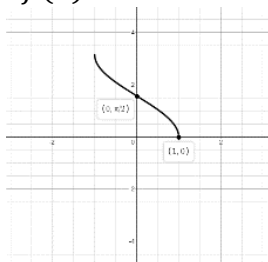
$$f(x) = \tan x$$



$$f(x) = \sin^{-1} x$$



$$f(x) = \cos^{-1} x$$



$$f(x) = \tan^{-1} x$$

