

Transformations of the Graphs

$$y = f(x) \xrightarrow{\text{Transformation to}} y = a \cdot f(b(x - c)) + d$$

3.
1.

2.
4.

Follow the order of operations when we perform transformations.

1. If $c > 0$, it causes horizontal shift to the right c units.
If $c < 0$, it causes horizontal shift to the left $|c|$ units.
2. If $|b| > 1$, it causes horizontal compression, $|b|$ times much.
If $0 < |b| < 1$, it causes horizontal stretch, $|b|$ times much.

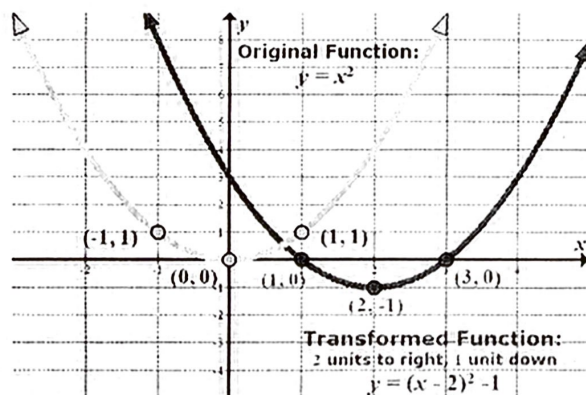
If $b < 0$, it causes reflection to the y-axis.

3. If $|a| > 1$, it causes vertical stretch, $|a|$ times much.
If $0 < |a| < 1$, it causes vertical compression, $|a|$ times much.

If $a < 0$, it causes reflection to the x-axis.

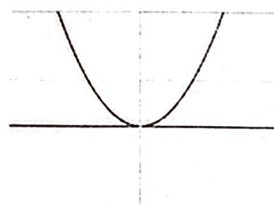
4. If $d > 0$, it causes vertical shift, d units up.
If $d < 0$, it causes vertical shift $|d|$ units down.

Ex) Given a graph of $y = x^2$,
sketch a graph of $y = (x - 2)^2 - 1$.

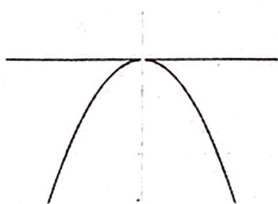


Good Graphs to Know

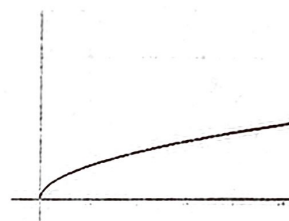
$$f(x) = x^2$$



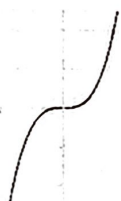
$$f(x) = -x^2$$



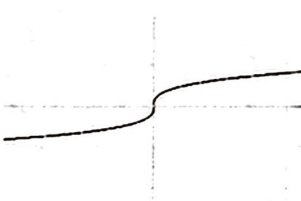
$$f(x) = \sqrt{x}$$



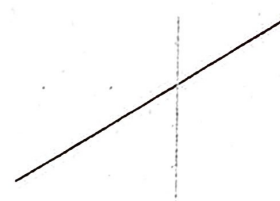
$$f(x) = x^3$$



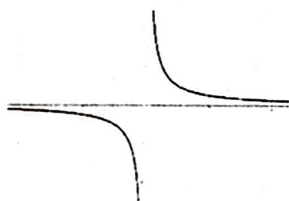
$$f(x) = \sqrt[3]{x}$$



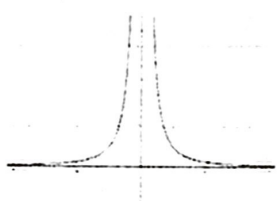
$$f(x) = x$$



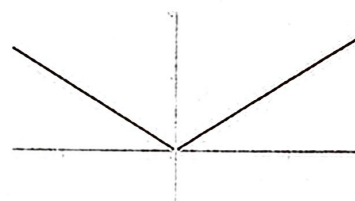
$$f(x) = \frac{1}{x}$$



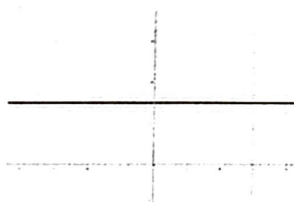
$$f(x) = \frac{1}{x^2}$$



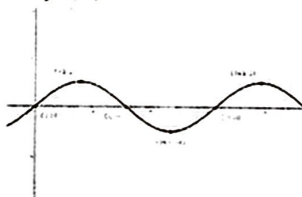
$$f(x) = |x|$$



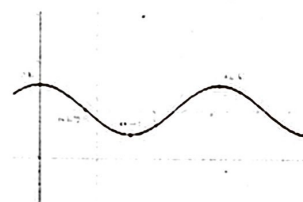
$$f(x) = c, c = 3$$



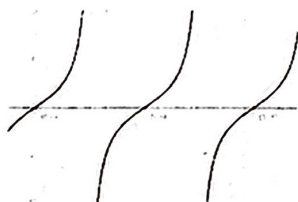
$$f(x) = \sin x$$



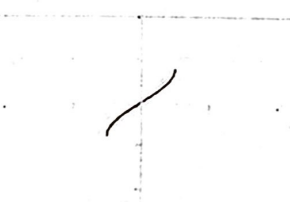
$$f(x) = \cos x$$



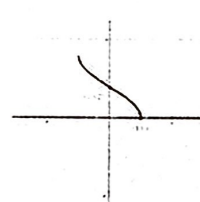
$$f(x) = \tan x$$



$$f(x) = \sin^{-1} x$$



$$f(x) = \cos^{-1} x$$



$$f(x) = \tan^{-1} x$$

