

DIFFERENTIATION RULES

General Formulas

$$1. \frac{d}{dx}(c) = 0$$

$$3. \frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$5. \frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) \quad (\text{Product Rule})$$

$$7. \frac{d}{dx}f(g(x)) = f'(g(x))g'(x) \quad (\text{Chain Rule})$$

$$2. \frac{d}{dx}[cf(x)] = cf'(x)$$

$$4. \frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

$$6. \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad (\text{Quotient Rule})$$

$$8. \frac{d}{dx}(x^n) = nx^{n-1} \quad (\text{Power Rule})$$

Exponential and Logarithmic Functions

$$9. \frac{d}{dx}(e^x) = e^x$$

$$11. \frac{d}{dx} \ln|x| = \frac{1}{x}$$

$$10. \frac{d}{dx}(a^x) = a^x \ln a$$

$$12. \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

Trigonometric Functions

$$13. \frac{d}{dx}(\sin x) = \cos x$$

$$16. \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$14. \frac{d}{dx}(\cos x) = -\sin x$$

$$17. \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$15. \frac{d}{dx}(\tan x) = \sec^2 x$$

$$18. \frac{d}{dx}(\cot x) = -\csc^2 x$$

Inverse Trigonometric Functions

$$19. \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$22. \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$20. \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$23. \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$21. \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$24. \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

Hyperbolic Functions

$$25. \frac{d}{dx}(\sinh x) = \cosh x$$

$$28. \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$26. \frac{d}{dx}(\cosh x) = \sinh x$$

$$29. \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$27. \frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$30. \frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

Inverse Hyperbolic Functions

$$31. \frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$

$$34. \frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

$$32. \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

$$35. \frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$33. \frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$$

$$36. \frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$$

TABLE OF INTEGRALS

Basic Forms

1. $\int u \, dv = uv - \int v \, du$

2. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

3. $\int \frac{du}{u} = \ln |u| + C$

4. $\int e^u \, du = e^u + C$

5. $\int a^u \, du = \frac{a^u}{\ln a} + C$

6. $\int \sin u \, du = -\cos u + C$

7. $\int \cos u \, du = \sin u + C$

8. $\int \sec^2 u \, du = \tan u + C$

9. $\int \csc^2 u \, du = -\cot u + C$

10. $\int \sec u \tan u \, du = \sec u + C$

11. $\int \csc u \cot u \, du = -\csc u + C$

12. $\int \tan u \, du = \ln |\sec u| + C$

13. $\int \cot u \, du = \ln |\sin u| + C$

14. $\int \sec u \, du = \ln |\sec u + \tan u| + C$

15. $\int \csc u \, du = \ln |\csc u - \cot u| + C$

16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C, \quad a > 0$

17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$

18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$

19. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$

20. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$

Forms Involving $\sqrt{a^2 + u^2}$, $a > 0$

21. $\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$

22. $\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$

23. $\int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$

24. $\int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$

25. $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$

26. $\int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$

27. $\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$

28. $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$

29. $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$

TABLE OF INTEGRALS

Forms Involving $\sqrt{a^2 - u^2}$, $a > 0$

$$1. \int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

$$1. \int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$2. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$3. \int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \sin^{-1} \frac{u}{a} + C$$

$$4. \int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

$$5. \int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$6. \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$$

$$7. \int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$$

$$8. \int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

Forms Involving $\sqrt{u^2 - a^2}$, $a > 0$

$$9. \int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$0. \int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$1. \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$$

$$2. \int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$$

$$3. \int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$$

$$4. \int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$$

$$5. \int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

$$6. \int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$$

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Forms Involving $a + bu$

$$47. \int \frac{u \, du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$$

$$48. \int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$$

$$49. \int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$$

$$50. \int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$$

$$51. \int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln |a + bu| + C$$

$$52. \int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$$

$$53. \int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3} \left(a + bu - \frac{a^2}{a + bu} - 2a \ln |a + bu| \right) + C$$

$$54. \int u \sqrt{a + bu} \, du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{3/2} + C$$

$$55. \int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$$

$$56. \int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2}{15b^3} (8a^2 + 3b^2u^2 - 4abu) \sqrt{a + bu} + C$$

$$57. \int \frac{du}{u \sqrt{a + bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, \quad \text{if } a > 0$$

$$= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bu}{-a}} + C, \quad \text{if } a < 0$$

$$58. \int \frac{\sqrt{a + bu}}{u} \, du = 2\sqrt{a + bu} + a \int \frac{du}{u \sqrt{a + bu}}$$

$$59. \int \frac{\sqrt{a + bu}}{u^2} \, du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2} \int \frac{du}{u \sqrt{a + bu}}$$

$$60. \int u^n \sqrt{a + bu} \, du = \frac{2}{b(2n + 3)} \left[u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} \, du \right]$$

$$61. \int \frac{u^n \, du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)} \int \frac{u^{n-1} \, du}{\sqrt{a + bu}}$$

$$62. \int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$$

TABLE OF INTEGRALS

Trigonometric Forms

$$\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$$

$$\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

$$\int \tan^2 u \, du = \tan u - u + C$$

$$\int \cot^2 u \, du = -\cot u - u + C$$

$$\int \sin^3 u \, du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$$

$$\int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$$

$$\int \tan^3 u \, du = \frac{1}{2}\tan^2 u + \ln |\cos u| + C$$

$$\int \cot^3 u \, du = -\frac{1}{2}\cot^2 u - \ln |\sin u| + C$$

$$\int \sec^3 u \, du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln |\sec u + \tan u| + C$$

$$\int \csc^3 u \, du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln |\csc u - \cot u| + C$$

$$\int \sin^n u \, du = -\frac{1}{n}\sin^{n-1}u \cos u + \frac{n-1}{n}\int \sin^{n-2}u \, du$$

$$\int \cos^n u \, du = \frac{1}{n}\cos^{n-1}u \sin u + \frac{n-1}{n}\int \cos^{n-2}u \, du$$

$$\int \tan^n u \, du = \frac{1}{n-1}\tan^{n-1}u - \int \tan^{n-2}u \, du$$

Inverse Trigonometric Forms

$$\int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1-u^2} + C$$

$$\int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1-u^2} + C$$

$$\int \tan^{-1} u \, du = u \tan^{-1} u - \frac{1}{2}\ln(1+u^2) + C$$

$$\int u \sin^{-1} u \, du = \frac{2u^2-1}{4}\sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$\int u \cos^{-1} u \, du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$76. \int \cot^n u \, du = \frac{-1}{n-1}\cot^{n-1}u - \int \cot^{n-2}u \, du$$

$$77. \int \sec^n u \, du = \frac{1}{n-1}\tan u \sec^{n-2}u + \frac{n-2}{n-1}\int \sec^{n-2}u \, du$$

$$78. \int \csc^n u \, du = \frac{-1}{n-1}\cot u \csc^{n-2}u + \frac{n-2}{n-1}\int \csc^{n-2}u \, du$$

$$79. \int \sin au \sin bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$$

$$80. \int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$$

$$81. \int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$$82. \int u \sin u \, du = \sin u - u \cos u + C$$

$$83. \int u \cos u \, du = \cos u + u \sin u + C$$

$$84. \int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$$

$$85. \int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$$

$$86. \int \sin^n u \cos^m u \, du = -\frac{\sin^{n-1}u \cos^{m+1}u}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2}u \cos^m u \, du \\ = \frac{\sin^{n+1}u \cos^{m-1}u}{n+m} + \frac{m-1}{n+m}\int \sin^n u \cos^{m-2}u \, du$$

$$92. \int u \tan^{-1} u \, du = \frac{u^2+1}{2}\tan^{-1} u - \frac{u}{2} + C$$

$$93. \int u^n \sin^{-1} u \, du = \frac{1}{n+1}\left[u^{n+1}\sin^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}}\right], \quad n \neq -1$$

$$94. \int u^n \cos^{-1} u \, du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1} u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}}\right], \quad n \neq -1$$

$$95. \int u^n \tan^{-1} u \, du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1} u - \int \frac{u^{n+1} du}{1+u^2}\right], \quad n \neq -1$$

TABLE OF INTEGRALS

Exponential and Logarithmic Forms

$$96. \int u e^{au} du = \frac{1}{a^2} (au - 1) e^{au} + C$$

$$97. \int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

$$98. \int e^{ax} \sin bu dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$$

$$99. \int e^{ax} \cos bu dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$$

$$100. \int \ln u du = u \ln u - u + C$$

$$101. \int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

$$102. \int \frac{1}{u \ln u} du = \ln |\ln u| + C$$

Hyperbolic Forms

$$103. \int \sinh u du = \cosh u + C$$

$$104. \int \cosh u du = \sinh u + C$$

$$105. \int \tanh u du = \ln \cosh u + C$$

$$106. \int \coth u du = \ln |\sinh u| + C$$

$$107. \int \operatorname{sech} u du = \tan^{-1} |\sinh u| + C$$

$$108. \int \operatorname{csch} u du = \ln |\tanh \frac{1}{2} u| + C$$

$$109. \int \operatorname{sech}^2 u du = \tanh u + C$$

$$110. \int \operatorname{csch}^2 u du = -\coth u + C$$

$$111. \int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$$

$$112. \int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$$

Forms Involving $\sqrt{2au - u^2}$, $a > 0$

$$113. \int \sqrt{2au - u^2} du = \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$114. \int u \sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$115. \int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$116. \int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$117. \int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$118. \int \frac{u du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$119. \int \frac{u^2 du}{\sqrt{2au - u^2}} = -\frac{(u+3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + C$$

$$120. \int \frac{du}{u \sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$$

Inverse Trig, Hyperbolic, and Inverse Hyperbolic Derivative and Integral Formulas

TABLE 7.3 Derivatives of the inverse trigonometric functions

- $\frac{d(\sin^{-1} u)}{dx} = \frac{du/dx}{\sqrt{1-u^2}}, \quad |u| < 1$
- $\frac{d(\cos^{-1} u)}{dx} = -\frac{du/dx}{\sqrt{1-u^2}}, \quad |u| < 1$
- $\frac{d(\tan^{-1} u)}{dx} = \frac{du/dx}{1+u^2}$
- $\frac{d(\cot^{-1} u)}{dx} = -\frac{du/dx}{1+u^2}$
- $\frac{d(\sec^{-1} u)}{dx} = \frac{du/dx}{|u|\sqrt{u^2-1}}, \quad |u| > 1$
- $\frac{d(\csc^{-1} u)}{dx} = \frac{-du/dx}{|u|\sqrt{u^2-1}}, \quad |u| > 1$

TABLE 7.4 Integrals evaluated with inverse trigonometric functions

The following formulas hold for any constant $a \neq 0$.

- $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \quad (\text{Valid for } u^2 < a^2)$
- $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C \quad (\text{Valid for all } u)$
- $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1}\left|\frac{u}{a}\right| + C \quad (\text{Valid for } |u| > a > 0)$

TABLE 7.7 Derivatives of hyperbolic functions

$$\begin{aligned}\frac{d}{dx}(\sinh u) &= \cosh u \frac{du}{dx} \\ \frac{d}{dx}(\cosh u) &= \sinh u \frac{du}{dx} \\ \frac{d}{dx}(\tanh u) &= \operatorname{sech}^2 u \frac{du}{dx} \\ \frac{d}{dx}(\coth u) &= -\operatorname{csch}^2 u \frac{du}{dx} \\ \frac{d}{dx}(\operatorname{sech} u) &= -\operatorname{sech} u \tanh u \frac{du}{dx} \\ \frac{d}{dx}(\operatorname{csch} u) &= -\operatorname{csch} u \coth u \frac{du}{dx}\end{aligned}$$

TABLE 7.8 Integral formulas for hyperbolic functions

$$\begin{aligned}\int \sinh u \, du &= \cosh u + C \\ \int \cosh u \, du &= \sinh u + C \\ \int \operatorname{sech}^2 u \, du &= \tanh u + C \\ \int \operatorname{csch}^2 u \, du &= -\coth u + C \\ \int \operatorname{sech} u \tanh u \, du &= -\operatorname{sech} u + C \\ \int \operatorname{csch} u \coth u \, du &= -\operatorname{csch} u + C\end{aligned}$$

TABLE 7.9 Identities for inverse hyperbolic functions

$$\begin{aligned}\operatorname{sech}^{-1} x &= \cosh^{-1} \frac{1}{x} \\ \operatorname{csch}^{-1} x &= \sinh^{-1} \frac{1}{x} \\ \coth^{-1} x &= \tanh^{-1} \frac{1}{x}\end{aligned}$$

TABLE 7.10 Derivatives of inverse hyperbolic functions

- $$\begin{aligned}\frac{d(\sinh^{-1} u)}{dx} &= \frac{1}{\sqrt{1+u^2}} \frac{du}{dx} \\ \frac{d(\cosh^{-1} u)}{dx} &= \frac{1}{\sqrt{u^2-1}} \frac{du}{dx}, \quad u > 1 \\ \frac{d(\tanh^{-1} u)}{dx} &= \frac{1}{1-u^2} \frac{du}{dx}, \quad |u| < 1 \\ \frac{d(\coth^{-1} u)}{dx} &= \frac{1}{1-u^2} \frac{du}{dx}, \quad |u| > 1 \\ \frac{d(\operatorname{sech}^{-1} u)}{dx} &= \frac{-du/dx}{u\sqrt{1-u^2}}, \quad 0 < u < 1 \\ \frac{d(\operatorname{csch}^{-1} u)}{dx} &= \frac{-du/dx}{|u|\sqrt{1+u^2}}, \quad u \neq 0\end{aligned}$$

TABLE 7.11 Integrals leading to inverse hyperbolic functions

- $\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C, \quad a > 0$
- $\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C, \quad u > a > 0$
- $\int \frac{du}{a^2 - u^2} = \begin{cases} \frac{1}{a} \tanh^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 < a^2 \\ \frac{1}{a} \coth^{-1}\left(\frac{u}{a}\right) + C & \text{if } u^2 > a^2 \end{cases}$
- $\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C, \quad 0 < u < a$
- $\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C, \quad u \neq 0 \text{ and } a > 0$

Expressing Inverse Hyperbolic Functions in terms of Logarithms (from p. 543)

$$\begin{aligned}\sinh^{-1} x &= \ln\left(x + \sqrt{x^2 + 1}\right), \quad -\infty < x < \infty \\ \cosh^{-1} x &= \ln\left(x + \sqrt{x^2 - 1}\right), \quad x \geq 1 \\ \tanh^{-1} x &= \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right), \quad |x| < 1 \\ \operatorname{sech}^{-1} x &= \ln\left(\frac{1 + \sqrt{1-x^2}}{x}\right), \quad 0 < x \leq 1 \\ \operatorname{csch}^{-1} x &= \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right), \quad x \neq 0 \\ \coth^{-1} x &= \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right), \quad |x| > 1\end{aligned}$$