STRATEGIES FOR SOLVING FIRST ORDER DIFFERENTIAL EQUATIONS

1. Determine whether the equation is separable, linear, and/or exact. If so, write it in the proper form:

NAME	FORM	METHOD FOR SOLVING
separable equations	$\frac{dy}{dx} = g(x)p(y)$	1) Multiply by dx and the reciprocal of $p(y)$. 2) Integrate both sides. 3) Solve for y if possible or for C otherwise.
linear equations	$\frac{dy}{dx} + P(x)y = Q(x)$	1) Multiply by the integrating factor $\mu(x) = e^{\int P(x)dx}$. 2) Rewrite the left side as $\frac{d}{dx}[\mu(x)y]$. 3) Integrate both sides. 4) Divide by $\mu(x)$.
exact equations	$M(x,y)dx + N(x,y)dy = 0$ where $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	1) Find $F(x,y) = \int M(x,y)dx + g(y)$. 2) Take the partial derivative with respect to y. 3) Set $g'(y) =$ whatever is missing from $N(x,y)$. 4) Integrate to obtain $g(y)$. 5) The solution is $F(x,y) = C$.

NOTE: An equivalent method for solving exact equations is to find $F(x,y) = \int N(x,y)dy + h(x)$, take the partial derivative with respect to x, set $h'(x) =$ whatever is missing from $M(x,y)$, integrate to obtain $h(x)$, and write the solution $F(x,y) = C$.

NOTE: In the process of solving a separable equation, solve $p(y) = 0$ before multiplying both sides by its reciprocal; for any c such that $p(c) = 0$, $y \equiv c$ is a solution that must be accounted for.

2. Check whether a substitution will transform the equation into one of the above:

NAME	EQUATION	SUBSTITUTION	RESULT
homogeneous equations	$\dfrac{dy}{dx}=G\left(\dfrac{y}{x}\right)$	$y=vx$ $dy=vdx+xdv$	separable equation
Bernoulli equations	$\dfrac{dy}{dx}+P(x)y=Q(x)y^{n}$ where $n\neq 0$ and $n\neq 1$	$v=y^{1-n}$ $\dfrac{dv}{dx}=(1-n)y^{-n}\dfrac{dy}{dx}$	linear equation
$ax+by$ equations	$\dfrac{dy}{dx}=G(ax+by)$	$z=ax+by$ $\dfrac{dz}{dx}=a+b\dfrac{dy}{dx}$	separable equation

NOTE: It is not necessary to write homogeneous equations in the specified form. To test for homogeneity, substitute tx and ty for x and y respectively, and show that the variable t divides out completely.

NOTE: The actual substitution for homogeneous equations is $v=\dfrac{y}{x}$ and the corresponding $\dfrac{dv}{dx}$; however, $y=vx$ and $dy=vdx+xdv$ are more convenient for computation.

NOTE: When solving a Bernoulli equation, divide both sides by y^{n} to get $y^{-n}\dfrac{dy}{dx}+P(x)y^{1-n}=Q(x)$. This form is more convenient for substitution: $\dfrac{1}{1-n}\cdot\dfrac{dv}{dx}+P(x)v=Q(x)$.