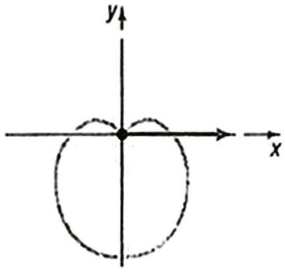
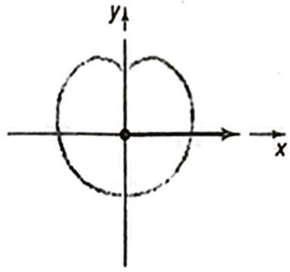
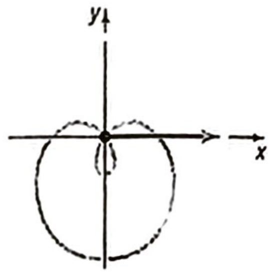
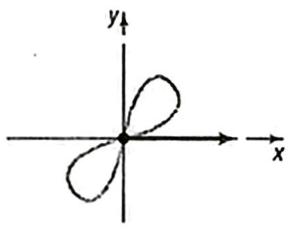
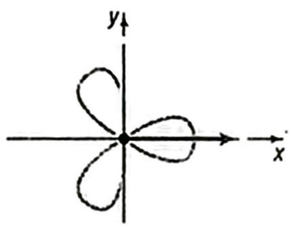
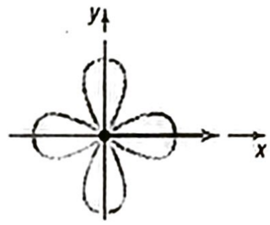
Other Equations

Name	Cardioid	Limaçon without inner loop	Limaçon with inner loop
Polar equations	$r = a \pm a\cos\theta,\ \ a > 0$ $r = a \pm a\sin\theta,\ \ a > 0$	$r = a \pm b\cos\theta,\ \ 0 < b < a$ $r = a \pm b\sin\theta,\ \ 0 < b < a$	$r = a \pm b\cos\theta,\ \ 0 < a < b$ $r = a \pm b\sin\theta,\ \ 0 < a < b$
Typical graph			
Name	Lemniscate	Rose with three petals	Rose with four petals
Polar equations	$r^2 = a^2\cos(2\theta),\ \ a \neq 0$ $r^2 = a^2\sin(2\theta),\ \ a \neq 0$	$r = a\sin(3\theta),\ \ a > 0$ $r = a\cos(3\theta),\ \ a > 0$	$r = a\sin(2\theta),\ \ a > 0$ $r = a\cos(2\theta),\ \ a > 0$
Typical graph			

Complex Numbers and De Moivre's Theorem

Polar form of a complex number
If $z = x + yi$, then $z = r(\cos\theta + i\sin\theta)$,
where $r = |z| = \sqrt{x^2 + y^2}$, $\sin\theta = \dfrac{y}{r}$, $\cos\theta = \dfrac{x}{r}$, $0 \le \theta < 2\pi$.

De Moivre's Theorem
If $z = r(\cos\theta + i\sin\theta)$, then $z^n = r^n\left[\cos(n\theta) + i\sin(n\theta)\right]$,
where $n \ge 1$ is a positive integer.

nth root of a complex number $w = r(\cos\theta_0 + i\sin\theta_0)$

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right)\right], \quad k = 0, \ldots, n-1,$$

where $n \ge 2$ is an integer

Vectors	A quantity having magnitude and direction; equivalent to a directed line segment $\overrightarrow{PQ}$
Position vector	A vector whose initial point is at the origin
Unit vector	A vector whose magnitude is 1
Direction angle of a vector **v**	The angle α, $0° \leq \alpha < 360°$, between **i** and **v**
Dot product	If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$, then $\mathbf{v} \cdot \mathbf{w} = a_1a_2 + b_1b_2$.
Angle θ between two nonzero vectors **u** and **v**	$\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\lVert\mathbf{u}\rVert\,\lVert\mathbf{v}\rVert}, 0 \leq \theta \leq \pi$

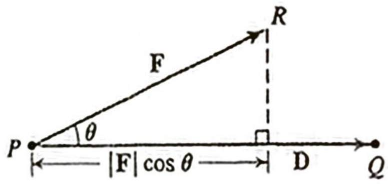
CALCULATING PROJECTIONS

The **projection of u onto v** is the vector $\text{proj}_{\mathbf{v}}\,\mathbf{u}$ given by

$$\text{proj}_{\mathbf{v}}\,\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$$

If the vector **u** is resolved into $\mathbf{u}_1$ and $\mathbf{u}_2$, where $\mathbf{u}_1$ is parallel to **v** and $\mathbf{u}_2$ is orthogonal to **v**, then

$$\mathbf{u}_1 = \text{proj}_{\mathbf{v}}\,\mathbf{u} \qquad \text{and} \qquad \mathbf{u}_2 = \mathbf{u} - \text{proj}_{\mathbf{v}}\,\mathbf{u}$$



$$W = \text{force} \times \text{distance} = (|\mathbf{F}|\cos\theta)|\mathbf{D}|$$

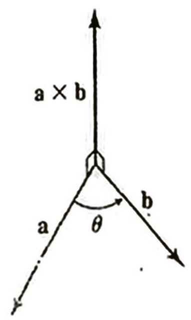
THE CROSS PRODUCT

If $\mathbf{a} = \langle a_1, a_2, a_3\rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3\rangle$ are three-dimensional vectors, then the **cross product** of **a** and **b** is the vector

$$\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1\rangle$$

CROSS PRODUCT THEOREM

The vector $\mathbf{a} \times \mathbf{b}$ is orthogonal (perpendicular) to both **a** and **b**.



Right-hand rule

LENGTH OF THE CROSS PRODUCT

If θ is the angle between **a** and **b** (so $0 \leq \theta \leq \pi$), then

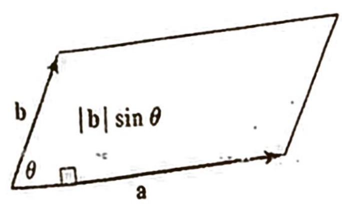
$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$$

In particular, two nonzero vectors **a** and **b** are parallel if and only if

$$\mathbf{a} \times \mathbf{b} = \mathbf{0}$$

AREA OF A PARALLELOGRAM

The length of the cross product $\mathbf{a} \times \mathbf{b}$ is the area of the parallelogram determined by **a** and **b**.



VOLUME OF A PARALLELEPIPED

The volume of the parallelepiped determined by the vectors **a**, **b**, and **c** is the magnitude of their scalar triple product:

$$V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$$

In particular, if the volume of the parallelepiped is 0, then the vectors **a**, **b**, and **c** are coplanar.