PHY 2049 Preparatory Workshop

with First Day Activity by Prof. Saludes

Preface

This workshop is designed to prepare students for PHY-2049 by demonstrating examples of effective problem-solving techniques and other tools for success in physics. For the student following through the calculations, note that all quantities written in this document are rounded to three significant figures. This is to emphasize the importance of using three significant figures when providing your answers. However, all of the final answers have been calculated using exact, unrounded values for all quantities in each problem. Therefore, final answers may appear to be inconsistent with the math shown, but these final answers are correct. Also note that the workshop demonstrates just one of many possible paths to a correct answer for each question - the student is advised to work to develop their own style for effective problem solving.

1. *A gas-powered electric generator produces a useful electrical power output of 450 kW. The generator has an overall efficiency of 43%.*
   a. *If the fuel (natural gas) has an energy density of 1150 Btu/ft^3, what volume flow rate must the fuel pump provide to the generator? Give your answer in ft3 per hour and m3 per second.*
   b. *If the fuel costs \$6.25 per 1000ft^3, what is the fuel-only cost per KWHR?*

a.

First, we break down the problem and attempt to identify the important details and relationships that we know.

<u>Relevant information:</u>

- $450\ kW\ output$
- $Efficiency\ =\ \frac{useful\ output}{input}\ =\ 43\%\ =\ 0.43$

Using these facts together, we can find necessary input power for the generator ($x\ kW$):

$$0.43 = \frac{output}{input} = \frac{450\ kW}{x\ \ kW}$$

$$x\ kW = \frac{450\ kW}{0.43} = 1{,}047\ kW$$

So, our input power must be $1{,}047\ kW$.

We now note the definition of kW:

$$kW\ =\ kiloWatts\ =\ \frac{kiloJoules}{second}$$

With this, we have the following:

$$1{,}047\ kW\ =\ 1{,}047\ \frac{kiloJoules}{second}$$

This means that 1,047 kJ (of energy - Joules are the SI unit of energy) are going into the motor every $second$. This quantity is energy flow rate (EFR), as kW are kJ (energy) per $second$ (time). Energy flowing into the motor is chemical potential energy stored in the fuel. So, energy flow rate in our situation is fuel volume flow rate or VFR ($\frac{volume}{time}$). Specifically, it is related in the following way:

$$EFR = \frac{energy}{time} = \frac{energy}{volume} \cdot \frac{volume}{time} = \rho \cdot (VFR)$$

Applying this, we have

$$1{,}047\ kW = \frac{1{,}047\ kJ}{second} = \frac{1{,}150\ Btu}{ft^3} \cdot \frac{x\ ft^3}{hour}$$

We first must note the following relationships:

- $1\ Btu\ =\ 1{,}054\ Joules$
- $1{,}000\ Btu\ =\ 1{,}054\ kiloJoules$
- $1\ hour\ =\ 3{,}600\ seconds$

We now convert units and arrange an equation.

$$\frac{1{,}047\ kJ}{second} = \left(\frac{1{,}150\ Btu}{ft^3} \cdot \frac{1{,}054\ kJ}{1{,}000\ Btu}\right) \cdot \left(\frac{x\ ft^3}{hour} \cdot \frac{1\ hour}{3{,}600\ seconds}\right)$$

We now solve this for $\frac{x\ ft^3}{hour}$.

$$\frac{1{,}047\ kJ}{second} \cdot \frac{3{,}600\text{ seconds}}{1\ hour} \cdot \frac{1{,}000\ Btu}{1{,}054\ kJ} \cdot \frac{1\ ft^3}{1{,}150\ Btu} = \frac{3{,}110\ ft^3}{hour}$$

So, we have found volume flow rate in $\frac{ft^3}{hour}$, and we now convert this to $\frac{m^3}{second}$.

To find in $\frac{m^3}{second}$, we use the following facts:

- $1\ m\ =\ 3.281\ feet$
- $1\ m^3 = (3.281\ ft)^3 = 35.32\ ft^3$

We now convert our volume flow rate using appropriate conversion factors:

$$\frac{3{,}110\ ft^3}{1\ hour} \cdot \frac{1\ m^3}{35.32\ ft^3} \cdot \frac{1\ hour}{3{,}600\ seconds} = \frac{2.45\ \cdot 10^{-2}\ m^3}{second}$$

b.

Since we need to find the price of fuel per kWh, we first see how much fuel must be put in for 1 kWh of energy to come out. We recall the efficiency of the generator to be 43%.

We use the following relationship:

$$Efficiency = 0.43 = \frac{useful\ output}{input}$$

Since we need a useful output of 1 kWh, we have

$$0.43 = \frac{1\ kWh}{x}$$

Calculating, the input x that will be needed is 2.33 kWh.

We need to see how much fuel (volume) will produce 2.33 kWh, so that we can calculate the price of it.

Recall the following:

- Energy density of fuel: $\frac{1{,}210\ kJ}{ft^3}$
- Cost of fuel: $\frac{\$6.25}{1{,}000\ ft^3}$
- $1\ hour = 3{,}600\ seconds$
- $1\ kWh = \frac{1\ kJ}{second} \cdot hour$

Applying these facts, we create the following equation to find the volume of fuel needed to provide 2.33 kWh of energy (this is the INPUT energy). On the left side, we have the energy from $x\ ft^3$ of fuel (in kJ) and the right side we have converted the 2.33 kWh into kJ as well.

$$\frac{1{,}210\ kJ}{ft^3} \cdot x\ ft^3 = 2.33\ kWh \cdot \frac{3{,}600\ seconds}{hour}$$

Solving this, we find that we need 6.91 ft^3 of fuel to create 1 kWh of energy (OUPUT). Since the cost is $\frac{\$6.25}{1{,}000\ ft^3}$ we have:

$$\frac{6.91\ ft^3}{kWh} \cdot \frac{\$6.25}{1{,}000\ ft^3} = \frac{\$x}{kWh}$$

We now have found the fuel-only cost per kWh is \$0.04, or 4¢.

2. *A force of* $(250 - 45t)\boldsymbol{x} + (200 + 5t^2)\boldsymbol{y}$ *N acts on 500 gram chihuahua (initially at rest) for 12 seconds.*
   a. *What is the final velocity of the chihuahua (speed and direction)?*
   b. *What is the final momentum of the chihuahua (magnitude and direction)?*
   c. *What is the total impulse applied to the chihuahua (magnitude and direction)?*
   d. *What is the total work done by the force?*

a.

We take a moment to consider this situation. First, we draw a free body diagram. After doing so, we realize that there was no friction or air resistance is mentioned here, so we neglect it. Also, we assume the chihuahua is on Earth, since there is nothing in the problem to suggest it is not. (In Professor Saludes' course, you may assume that the scenarios described are on Earth unless otherwise stated.) This means that we must consider the force of gravity on earth when considering the motion of the chihuahua.

Physics Tip: Free body diagrams are extremely powerful tools for analyzing complex situations with forces. When attempting to analyze motion or any other physical situation, it can be extremely beneficial to construct a free body diagram. The general idea of a free body diagram is to create a picture of a situation with bodies and the forces acting upon them in order to better analyze the result of a given combination of forces. Here are some tips to draw effective free body diagrams which will help you more easily understand a situation and find a solution:

- Draw the bodies in your scenario as a simple circle or square and label each body in your diagram (simply write a letter on top of the body).
- Draw all forces as a solid arrow with the tail on the object being acted upon, and the head in the direction that the force is pushing or pulling.
- Clearly label all arrows to indicate what they represent. You can denote forces with $\vec{F}$ and use an appropriate subscript to indicate which force an arrow represents.
- Add arrows to represent all forces which are involved in your scenario.
- Check for "hidden" forces (forces such as gravity, normal force, friction, and air resistance), and draw in these forces too.
- If there are relevant velocities for the bodies in the diagram, denote them in such a way that they are not mistaken for a force (use a solid arrow which does not touch the body, and which is clearly labelled as a velocity).
- If necessary, draw components of forces using a dotted line for the arrow rather than a solid line. Labelling these arrows as well to distinguish which force and which component of the force they represent.
- If necessary, label angles at which forces act on the bodies.
- If there are any walls or ceilings in the scenario, draw them using dotted lines for these so they are not mistaken for forces.

We convert the mass of the chihuahua to kilograms:

$$\frac{500\ g}{chihuahua} \cdot \frac{1\ kg}{1{,}000\ g} = \frac{0.5\ kg}{chihuahua}$$

We first calculate the x-component of the final velocity of the chihuahua using the following general integral:

$$\int_{t_i}^{t_f} a\ \cdot dt = v_f - v_i$$

The only acceleration on the chihuahua in the x-axis is from the force applied to it: $\vec{F}_{xe} = 250 - 45t$, and $v_{xi} = 0$ since the chihuahua began at rest.

We recall the following:

- Newton's 2nd Law: $\Sigma\,\vec{F} = \mathrm{m} \cdot \vec{a}$.
- m (mass of chihuahua) = 0.5 kg
- $\vec{F}_{xe} = 250 - 45t$ (subscript "e" to denote the external force acting on the chihuahua)

NOTE: We use subscripts to denote the axis which our quantity (force, acceleration, velocity) is referring to, and we also use subscripts (*i* and *f*) to denote whether the quantities are initial or final values. We may also use other subscripts on the capital F (for forces) to denote which force we are talking about.

So, we can find the acceleration of the chihuahua along the x-axis.

$$\vec{F}_{xe} = 250 - 45t = 0.5\ kg \cdot \vec{a}_x$$

$$\vec{a}_x = 500 - 90t$$

We now have acceleration of the chihuahua along the x-axis as a function of time and can use our integral, since we know the chihuahua is acted on by the force for 12 seconds.

$$\int_0^{12} (500 - 90t)\ dt = \vec{v}_{xf} - \vec{v}_{xi} = \vec{v}_{xf} - 0 = \vec{v}_{xf} = -480\ \frac{m}{s}$$

Now we do the same for the y-component.

We have $\Sigma\overrightarrow{F_y} = 0.5\ kg \cdot \vec{a}_y$, and the chihuahua has both gravity and the given external force acting on it.

For bodies near the surface of the Earth, acceleration due to gravity is about 9.81 $\frac{m}{s^2}$. However, gravity pulls the object down, so we must make sure to use -9.81 $\frac{m}{s^2}$, to account for this. So, the force of gravity here can be written as follows: $\vec{F}_g = -9.81\ \frac{m}{s^2} \cdot 0.5\ kg$. We will treat this force as constant, as we assume that the chihuahua will not travel extremely far away from the surface of the Earth and therefore any changes in the force of gravity will be negligible.

We also have the y-component of the external force $\vec{F}_{ye} = 200 + 5t^2 N$, and $\Sigma\overrightarrow{F_y} = 0.5\ kg \cdot \vec{a}_y \frac{m}{s^2}$.

In the case of the y-component forces, we have $\Sigma\vec{F}_y = \left(9.81\ \frac{m}{s^2} \cdot 0.5\ kg\right) + (200 + 5t^2)\text{N}$, and therefore

$$0.5\ kg \cdot \vec{a}_y = \left(-9.81\ \frac{m}{s^2} \cdot 0.5\ kg\right) + (200 + 5t^2)\text{N}.$$

Solving this for $\vec{a}_y$, we have $\vec{a}_y = 390 + 10t^2$. We now apply our integral over the 12 $second$ time span to find the y-component of the chihuahua's final velocity.

That's a very fast chihuahua. Fun fact: This chihuahua will be at a height of 45,360 $meters$ by the time it reaches a (y-component) speed of 10,440 $\frac{m}{s}$. Another fun fact: Professor Saludes is NOT a fan of chihuahuas.

$$\int_0^{12} (390 + 10t^2)\, dt = \vec{v}_{yf} - \vec{v}_{yi} = \vec{v}_{yf} - 0 = \vec{v}_{yf} = 10{,}440\frac{m}{s}$$

We now know the x and y components of the chihuahua, and can use these to calculate the total speed, as well as the direction of the chihuahua.

To find the final total velocity, we recall "polar form" (r, θ) from Precalculus. The following equations will allow us to find the magnitude (r) which represents v_f the final total velocity of the chihuahua, and direction θ (relative to the horizontal) of the chihuahua's final velocity.

- $r = \sqrt{x^2 + y^2}$
- $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

NOTE: When the x-component is negative, we must add 180 degrees to whatever answer we get from the inverse tangent function. This is because inverse tangent only returns values in the right half of the unit circle (where the "x-component" cosine, is positive).

We apply these formulas where the x and y mentioned above are $\vec{v}_{xf}$ and $\vec{v}_{yf}$, the final velocity x and y components of the velocity of the chihuahua.

$$\|v_f\| = \sqrt{{\vec{v}_{xf}}^2 + {\vec{v}_{yf}}^2} = \sqrt{(-480\frac{m}{s})^2 + (10{,}440\frac{m}{s})^2} = 10{,}500\frac{m}{s}$$

$$\theta = \tan^{-1}\left(\frac{\vec{v}_{yf}}{\vec{v}_{xf}}\right) = \tan^{-1}\left(\frac{10{,}440}{-480}\right) + 180° = 92.6°$$

So, the chihuahua's final total speed (the magnitude of its final velocity, $\|v_f\|$) is $10{,}500\frac{m}{s}$, and the chihuahua is travelling at an angle of 92.6° relative to the horizontal.

b.

We remind ourselves the definition of momentum:

$$\vec{p} = m \cdot \vec{v}$$

Using the previous results $m = 0.5\ kg$ and $v_f = 10{,}500\ \frac{m}{s}$, we calculate the final momentum of the chihuahua:

$$\vec{p} = 0.5\ kg \cdot\ 10{,}500\frac{m}{s} = \ 5{,}250\ \frac{N}{(\frac{m}{s})}$$

The direction of the momentum vector is determined by the direction of the velocity. This is because mass is a scalar quantity, so the product of mass and velocity results in a vector with the same direction as the velocity: 92.6° relative to the horizontal.

c.

Here we remind ourselves the definition of impulse: change in momentum. That is, $\Delta\vec{p} = \ \overrightarrow{p_f} - \overrightarrow{p_i}$. Noting that we already have found our final momentum $\vec{p}_f = \ 5{,}250\frac{N}{\frac{m}{s}}$, and that the chihuahua was at rest initially, we have

$$(\vec{p}_i = m \cdot \vec{v}_i = 0.5\ kg\ \cdot \frac{0\ m}{s} = \frac{0\ N}{\frac{m}{s}})$$

And so the impulse of the chihuahua is:

$$\Delta\vec{p} = \ \vec{p}_f - \vec{p}_i = \ \ 5{,}250\frac{N}{\frac{m}{s}} - \ 0\frac{N}{\frac{m}{s}} = \ 5{,}250\frac{N}{\frac{m}{s}}$$

<u>NOTE:</u> We could alternatively use the impulse integral: $\Delta\vec{p} = \int_{t_i}^{tf} \vec{F} \cdot dt$ to solve this part of the problem. Make sure you are familiar with the integrals for each topic as you go through class, as they can simplify problems that would otherwise be difficult.

d.

Here we will use the work energy theorem: $W = \ \Delta E = E_f - E_i$. We note that the chihuahua in its final position has both kinetic and gravitational potential energy.

First, we want to find E_f, so we will calculate the final gravitational potential energy and kinetic energy of the chihuahua at the end of the period over which the external force is applied. For gravitational potential energy, we need to find the final height of the chihuahua. We leave this calculation to the students. To find final height of the chihuahua, remind yourself of the following:

$$\Delta y = \ \int_{t_i}^{t_f} \vec{v}_y(t) \cdot dt \text{ and } \frac{d}{dt}\vec{a}_y(t) = \ \vec{v}_y(t).$$

The final height of the chihuahua (as mentioned in the "fun fact" section) is 45,360 meters. So, the final gravitational potential energy of the chihuahua is

$$U_f = m \cdot g \cdot h_f = 0.5\ kg\ \cdot 9.81 \frac{m}{s^2} \cdot 45{,}360\ m = 223\ kJ.$$

We then find the final kinetic energy of the chihuahua:

$$KE_f = \frac{1}{2} m \cdot \vec{v}_f^{\ 2} = \frac{1}{2} \cdot 0.5\ kg\ \cdot \left(10{,}500\ \frac{m}{s}\right)^2 = 27.3\ MJ.$$

Now we can find the final total energy of the chihuahua,

$$E_f = U_f + KE_f = 223\ KJ + 27.3\ MJ = 27.5\ MJ.$$

We now calculate the initial energy of the chihuahua. The chihuahua was at rest initially, so it has no kinetic energy. We also assume it was on the ground, and therefore had no gravitational potential energy. So, the initial energy of the chihuahua is $0\ J$.

To find the work done on the chihuahua by the external force, we use the work-energy theorem mentioned earlier:

$$W = \Delta E = E_f - E_i = 27.5\ MJ - 0J = 27.5\ MJ$$

So, the <u>work done on the chihuahua by the force is 27.5 MJ.</u>

3. *A motorcycle weighs 680 pounds total, including the very handsome rider. The two wheels each have a mass of 18 kg, a diameter of 24.5 inches and an effective moment of inertia of ¾ MR^2. If the motorcycle can accelerate from 0 - 60 MPH in 3.20 seconds, what is the total output power being provided by the engine?*

Physics Tip: Many times throughout this course (and in your life or chosen area of practice) you will encounter problems where it is not immediately evident which course of action will be best, or even which pieces of information you may need to solve a problem. A very important skill to develop to succeed in the problems you encounter is to create some method of analyzing a complex situation in such a way that you can attempt to find a solution. In the explanations provided, you will see just a few of many viable strategies that may help you succeed in such endeavors, and it is critical that you work to develop a technique that helps *you*.

To begin this problem, we must focus on the question that is asked of us – the *output* **power** provided by the engine of the motorcycle.

We remind ourselves the definition of power: $\mathrm{P} = \frac{\Delta E}{\Delta t}$.

Now we realize that we can find the output power provided by the engine of the motorcycle if we can find the total energy added to the system (by the engine), ΔE over some period of time, Δt.

We notice that we are given the time needed for the motorcycle to accelerate to a speed of $60\ MPH$ from rest: $3.20\ seconds$. We may use this information to find to find the linear kinetic energy of the system after $3.20\ seconds$ if we know the mass of the system.

We also must note that the engine provides rotational kinetic energy to the wheels, and this must also be considered in our calculation of ΔE.

We will begin by finding the linear kinetic energy of the system after 3.20 seconds:

$$KE_f = \frac{1}{2} \cdot m \cdot \vec{v}_f^{\ 2}$$

We know the final velocity $\vec{v}_f = 60\ MPH$, but we also must find the *mass* of the system, as we are only given the weight.

We remind ourselves that weight is a force- the force with which gravity pulls an object down. We assume the motorcycle is on Earth, where force due to gravity (near the surface of the Earth) is given by $w = m \cdot g$.

Then we have $680\ lbs = m\ kg \cdot 9.81\ \frac{m}{s^2}$. We need the weight to be converted into the standard unit of force, Newtons, so we use the following relationship: $1\ lb = 4.448\ N$.

$$\frac{680\ lbs}{motorcylce} \cdot \frac{4.448\ N}{1lb} = 3{,}002\ N$$

We now have $3{,}002\ N = m\ kg\ \cdot 9.81 \frac{m}{s^2}$, and solving for m yields $m = 308\ kg$.

We also must convert the final velocity of the motorcycle from MPH to meters per second.

We note the following:

- $1\ hour\ =\ 3{,}600\ seconds$
- $1\ mile\ =\ 1{,}609\ meters$

We then have:

$$\vec{v}_f = \frac{60\ miles}{1\ hour} \cdot \frac{1609\ meters}{1\ mile} \cdot \frac{1\ hour}{3600\ seconds} = 26.8\ \frac{m}{s}$$

So, the linear kinetic energy of the motorcycle and rider can be found:

$$KE_f = \frac{1}{2} \cdot m \cdot \vec{v}_f^{\ 2} = \frac{1}{2} \cdot 308\ kg\ \cdot \left(26.8 \frac{m}{s}\right)^2 = 111\ J$$

Now we try to find the rotational kinetic energy of the motorcycle's wheels. We recall the formula for rotational kinetic energy:

$$K_r = \frac{1}{2} I \omega^2$$

We know that both wheels will be spinning, and the rate at which the wheels spin is related to how fast the motorcycle travels on the ground. We recall the following formula explaining this relationship:

$$\|\vec{v}\| = r\omega$$

We realize we can find the maximum angular velocity of the wheel at the end of the $3.20\ seconds$ since we know the final speed of the motorcycle is $26.8\frac{m}{s}$, and we can calculate the radius of the wheels from the given diameter.

We are given that $d = 24.5\ inches$, so we use the following relationships to convert this length into standard units:

- $1\ inch = 2.54\ centimeters$
- $d = 2r$
- $100\ centimeters = 1\ meter$

We now convert this and find the radius:

$$r = \frac{d}{2} = \frac{24.5\ inches}{2} \cdot \frac{2.54\ cm}{1\ inch} \cdot \frac{1\ meter}{100\ cm} = 0.311\ meters$$

Physics Tip: The conversion factor we use ($\frac{2.54\ cm}{1\ inch}$) and the diameter of the wheel ($24.5\ inches$) are two very similar numbers. "Coincidences" such as these occur frequently on the exams in professor Saludes' course, and may occur in real life as well. For this reason, students should be prudent and avoid performing unnecessary calculations which do not directly aid the problem-solving process. In complex problems with many distinct quantities which are related by certain formulas, one useful strategy is to apply algebra to the relevant equations to solve for the desired variable BEFORE actually "plugging in" any of the given or calculated values. This leaves fewer opportunities for mistakes, and can occasionally reveal that the problem includes extraneous values which need not be used at all, unless the student does not properly apply algebra.

We now have $\vec{v}_f$ of the motorcycle and the radius r of both wheels, and we use this to find the angular velocity of the wheels with the formula $\|\vec{v}\| = r\omega$.

NOTE: The standard unit for angular velocity is radians per second. This means we need to keep an eye out to be sure we end up with the correct units – it may be tempting to give ω in Hertz because we see that the units end up appearing to be $\frac{cycles}{second}$. However, we are responsible for recognizing that angular velocity must have units of radians per second.

Fun Fact: Radians are a special kind of unit called a "dimensionless unit". In practice, this means that radians do not carry a dimension with them such as meters or liters. This fact means that we can "insert" radians as we deem necessary without needing to multiply by any kind of conversion factor. Why are radians a dimensionless unit? Radians have no dimension because they are a ratio: how many times a radius (some unit of length) fits along the circumference of a circle (another unit of length).

$$26.8\ \frac{m}{s} = 0.311\ m \cdot \omega_f \frac{radians}{second}$$

$$\omega_f = 86.2\ \frac{radians}{second}$$

Now that we have the angular velocity of the wheels, we can find the rotational kinetic energy of the wheels at the end of the 3.20 $second$ period of acceleration. We accomplish this using the previously mentioned relationships and facts:

- $K_r = \frac{1}{2}I\omega^2$ (per wheel)
- $I = \frac{3}{4}MR^2$
- $\omega_f = 86.2\ \frac{radians}{second}$
- $M = 18\ kg$
- $r = 0.311\ meters$

So, we find the total kinetic rotational energy of motorcycle is as follows:

$$2\ wheels \cdot \frac{K_{rf}}{wheel} = 2\left(\frac{1}{2}I{\omega_f}^2\right) = I{\omega_f}^2 = \frac{3}{4}MR^2{\omega_f}^2 = \frac{3}{4} \cdot 18\ kg\ \cdot\ (0.311\ m)^2 \cdot \left(86.2\ \frac{radians}{second}\right)^2$$
$$= 9{,}710\ J$$

Now that we have the total kinetic rotational energy of the motorcycle, we add it to the linear kinetic energy we found earlier to find the total change in energy in the 3.20 $seconds$:

$$\Delta E = E_f - E_i = (KE_f + K_{rf}) - (KE_i + K_{ri}) = (111\ J + 9{,}710\ J) - (0 + 0) = 9{,}820\ Joules = 9.82\ KJ$$

We now know the change in energy and change in time, and can therefore calculate the output power provided by the engine of the motorcycle:

$$\text{P} = \frac{\Delta E}{\Delta t} = \frac{9.82\ KJ}{3.20\ seconds} = 3.07\ KW$$

So, the output power provided by the engine of the motorcycle is 3.07 kW.

4. *Silver has a specific heat of 240 J/kg °C, a latent heat of fusion of 111 kJ/kg and a melting point of 962°C. Iron has specific heat of 450 J/kg °C, a latent heat of fusion of 138 kJ/kg and and a melting point of 1538°C. You put 750 grams of silver in a 1.25 kg iron pot, initially at 25°C, into an oven, and the silver is completely melted in 23 minutes. What is the minimum power of the heater in the oven?*

We first identify what we are asked to do in the problem. Here we must find the minimum power of the heater in the oven. Power is the rate of energy transfer ($P = \frac{\Delta E}{\Delta t}$) so we must find how much energy (ΔE) the oven transfers in some amount of time (Δt).

We pause and think of a way to progress in our problem. Then, we realize that for our situation, we know that some amount of energy will be transferred to the silver and iron over a period of 23 minutes, and this energy is enough to completely melt the silver. So, in order to find $P = \frac{\Delta E}{\Delta t}$ we just need to find the minimum energy (ΔE) that the oven transferred to the iron and silver.

Physics Tip: When analyzing a situation in physics, there tends to be more going on that meets the eye. Often it is appropriate to draw a free body diagram or other picture to understand and consider all possible factors in a scenario. One particularly important phenomenon for students to identify is when multiple bodies will or will not act as a single combined body. To help with this, try breaking it down into temporal "stages" (or possible realities) in which different rules will come into play.

For example, we can use temporal stages to analyze a situation where a rocket launches into the air with the help of boosters:
Stage 1: The rocket boosters are attached to the rocket (they act as a single combined body!) and providing an upwards force, while gravity and air resistance act in opposition to the boosters.
Stage 2: The rocket boosters have run out of fuel and now detach from the rocket (they no longer act as a combined body!). Meanwhile, the rocket continues to its maximum height.*
Stage 3: The rocket now falls back to Earth from its maximum height.
*-Here we would consider different "possible realities". With the rocket at maximum height, it may be in space or the Earth's atmosphere, and friction differs greatly between these two possible scenarios (similar idea as when we check if an object will experience kinetic friction or static friction).

In this problem, we first consider the stage when the silver inside the iron pot is outside the oven. We know that these are two different substances that will react differently to heat. However, over a period of 23 minutes, it is fair to treat the silver and iron as a "combined body" with regards to temperature. That is, because they will be in constant contact for a significant time, we expect them to both reach the same temperature by the time they leave the oven. So, we must also find the amount of energy transferred into the iron in order for it to reach the same final temperature as the silver.

Now we have a goal: find the energy needed to completely melt 750 grams of solid silver.

We do not know the exact final temperature of the silver and the iron pot. However, we only need to find the minimum energy transfer that would result in the silver being completely melted – this means the silver (and therefore the iron pot) will have a final temperature of at least $962°C$. We know that this is the minimum final temperature because it is the melting point of silver, which leaves the oven completely melted, and the iron remains in contact with the silver the entire time, so we expect it to have the same temperature.

We recall that energy is involved not only in changing the temperature of a substance, but also in phase changes. The relationships are described in the following equations:

- $Q = m \cdot c \cdot \Delta T$
- $Q = m \cdot L$

Here, m is the mass of a substance in kilograms, Q is the heat (energy transferred) in $Joules$, c is the specific heat of a substance in $\frac{Joules}{kg\cdot°C}$, L is the latent heat of fusion (L_f) or vaporization (L_v) in $\frac{Joules}{kg}$, and ΔT is the change in energy in °C (or Kelvin, because the kelvin scale is the same as Celsius aside from being offset by 273.18 degrees/Kelvin).

Physics Tip: When applying equations to for multiple distinct objects, it is important to use subscripts distinguish between the objects or quantities that they describe. Here, we have the general equations for Q, the amount of energy transferred to a substance to increase its temperature or change its phase. Here, we have three different Q's: the energy transfer to increase the temperature of the silver (Ag), change the silver's phase, and to increase the temperature of the iron (Fe). So, might distinguish between these by instead using $Q_{t\ Ag}$, $Q_{f\ Ag}$(subscript f for fusion), and $Q_{t\ Fe}$, to represent each of these respectively. We will use similar notation for the mass, specific heat, and latent heat of fusion of each substance. Often, there is no standard for attaching subscripts to variables. This means that you should pick a method of organization that works for *you*. Food for thought as we go through this problem: Why did we decide to not to use subscripts for the quantity ΔT?

So, we first will try to find the energy required to completely melt all the silver. We know that energy must be provided to take the silver up to melting temperature and allow the phase change. We will use the following equations and facts to find this energy:

- $Q_{t\ Ag} = m_{Ag} \cdot c_{Ag} \cdot \Delta T$
- $Q_{f\ Ag} = m_{Ag} \cdot L_{fusion\ Ag}$
- Silver's melting point: $T_{f\ min} = 962°C$
- $T_i = 25°C$
- $m_{Ag} = 0.75\ kg$
- $c_{ag} = \frac{240\ J}{kg\cdot°C}$
- $L_{f\ Ag} = \frac{111\ kJ}{kg}$

We first want to find the heat required for the temperature change. To do this, we need to calculate $\Delta T = T_f - T_i = 962°C - 25°C = 937°C$. We use this to find the heat required to heat the silver:

$$Q_{t\ Ag} = m_{Ag} \cdot c_{Ag} \cdot \Delta T = (0.75\ kg) \cdot \left(\frac{240\ J}{kg\ \cdot °C}\right) \cdot 937°C = 169\ kJ$$

Now we calculate the heat required for the silver to undergo fusion:

$$Q_{f\ Ag} = m_{Ag} \cdot L_{f\ Ag} = 0.75\ kg\ \cdot \frac{111\ kJ}{kg} = 83.3\ kJ$$

We now know the minimum energy provided by the oven to cause the silver to completely melt. The only other energy transfer we need to consider is the energy transferred to the iron pot. We use the following equations and facts:

- $Q_{t\ Fe} = m_{Fe} \cdot c_{Fe} \cdot \Delta T$
- $T_i = 25°C$
- $c_{Fe} = \frac{450\ J}{kg\cdot°C}$

- Iron has same minimum temperature as the silver, so $T_{f\ min} = 962℃$
- $m_{Fe} = 1.25\ kg$

We do not know the final temperature of the iron and silver, we only know that the temperature must be at least $962℃$. Since this is lower than the melting point of iron (1,538℃), we do not perform any calculations for phase change of iron. We now proceed to calculate the minimum necessary energy for this temperature change in the iron pot:

$$Q_{t\ Fe} = m_{Fe} \cdot c_{Fe} \cdot \Delta T = 1.25\ kg\ \cdot \left(\frac{450\ J}{kg\ \cdot ℃}\right) \cdot 937℃ = 527\ kJ$$

Now that we have found the minimum energy transfer to heat the iron and silver as well as melt the silver, we find their sum in order to calculate the minimum power of the heater:

$$\Delta E_{total} = Q_{t\ Ag} + Q_{fusion\ Ag} + Q_{t\ Fe} = 169\ kJ + 83.3\ kJ + 527\ kJ = 696\ kJ$$

Now we have the minimum energy transfer (ΔE_{total}) provided by the heater, and we know the time period (Δt) is 23 minutes, we use the following relationship to convert this time into standard units, and then find the minimum power of the heater:

- $1\ minute = 60\ seconds$
- $P_{min} = \frac{\Delta E_{total}}{\Delta t}$

So we have that the minimum power of the heater of the oven is as follows:

$$P_{min} = \frac{\Delta E_{total}}{\Delta t} = \frac{696\ kJ}{23\ minutes} \cdot \frac{1\ minute}{60\ seconds} = 504\ W$$

Therefore, the minimum power supplied by the oven heater is $504\ W$.