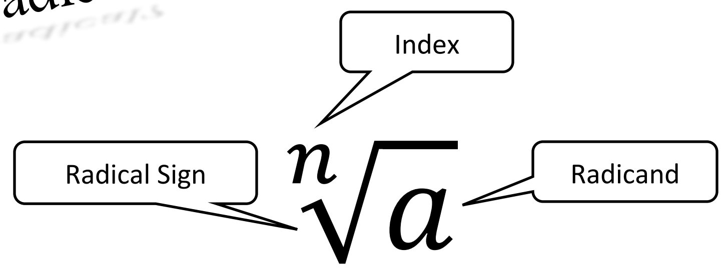
STEM Center (BLRC 200)
Academic Success Center
HCC Brandon Campus

Introduction to Radicals

What is a radical?

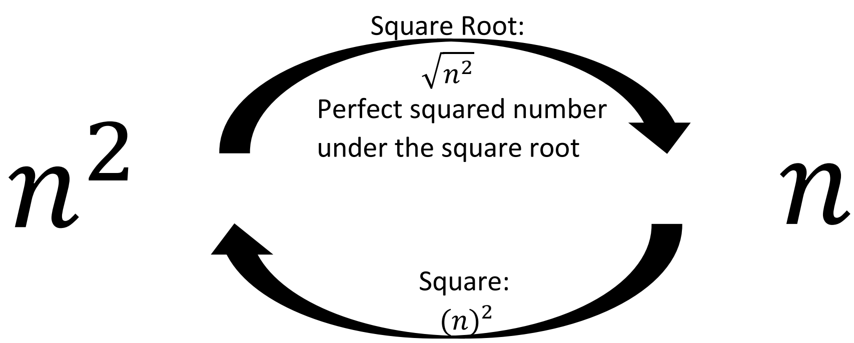


< Radical Expression >

The symbol $\sqrt{\ }$ that we use to denote the principal square root is called a radical or radical sign for any real number a, and integer $n \geq 2$.

"Roots" (or "radicals") are the "opposite" operation of applying exponents; we can "undo" a power with a radical, and we can "undo" a radical with a power.

If the radicand has a perfect n^{th} power, we can get rid of the radical. If the radicand has a perfect n^{th} power factors, each one of those factors can be out from the radical to be simplified.



For example,

$$2^2 = 4, \qquad so\ \sqrt{4} = 2$$

$$5^2 = 25, \qquad so\ \sqrt{25} = 5$$

Let's simplify $\sqrt{49}$.

Square Root:
$\sqrt{49} = \sqrt{7^2}$
Perfect squared number under the square root

$$49 \quad \rightleftarrows \quad 7$$

Square:
7^2

So, $\sqrt{49} = 7$.

Then, how can we simplify $\sqrt[3]{-27}$?

Cube Root:
$\sqrt[3]{-27} = \sqrt[3]{(-3)^3}$
Perfect cubed number under the cube root

$$-27 \quad \rightleftarrows \quad -3$$

Cube:
$(-3)^3$

So, $\sqrt[3]{-27} = -3$.

You Try!

Ex1) Simplify:

a. $\sqrt{64} = \sqrt{8 \cdot 8} = \sqrt{8^2} =$

b. $\sqrt{\frac{1}{81}} = \sqrt{\frac{1}{\quad} \cdot \frac{1}{\quad}} = \sqrt{\left(\frac{1}{\quad}\right)^2} =$

c. $-\sqrt[3]{0.008} = -\sqrt[3]{\quad \cdot \quad \cdot \quad} = -\sqrt[3]{(\quad)^3} =$

d. $\sqrt[5]{-x^5} =$

The Product and Quotient Rules for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, and n is an integer ($n \geq 2$), then

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \;\; and \;\; \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab},$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0 \;\; and \;\; \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}, b \neq 0.$$

Ex2) Let's simplify the following radical expressions.

a. $\sqrt{450} = \sqrt{5 \cdot 9 \cdot 10} = \sqrt{2 \cdot 3^2 \cdot 5^2} = 3 \cdot 5 \cdot \sqrt{2} = 15\sqrt{2}$

b. $\sqrt[3]{48x^6y^7} = \sqrt[3]{2 \cdot 2^3 \cdot 3 \cdot x^3 \cdot x^3 \cdot y^3 \cdot y^3 \cdot y} = 2 \cdot x \cdot x \cdot y \cdot y \cdot \sqrt[3]{2 \cdot 3 \cdot y} = 2x^2y^2\sqrt[3]{6y}$

You Try!

c. $\sqrt[4]{81a^9b^8} =$

d. $\sqrt[3]{\dfrac{16c^5d^7}{250c^2d^2}} =$

Is the square root of a negative number a real number such as $\sqrt{-25}$?

Is there a real number whose square is -25? **No**. Thus, $\sqrt{-25}$ is not a real number.

Under the square root or even root (if the index is even), the radicand should be ALWAYS a nonnegative number to be a real number.

Definition of the Principal n^{th} root of a real number a:

$\sqrt[n]{a} = b$ means that $b^n = a$.

If n, the index, is even, then a is nonnegtive $(a \geq 0)$ and b is also nonnegative $(b \geq 0)$. If n is odd, a and b can be any real numbers.

If the index n is an odd number, a root is said to be an odd root. Likewise, if the index n is an even number, a root is said to be an even root.

If n is odd, $\sqrt[n]{a^n} = a$.   ex) $\sqrt[3]{(-2)^3} = -2$

If n is even, $\sqrt[n]{a^n} = |a|$.   ex) $\sqrt[4]{(-2)^4} = |-2| = 2$ (or since $\sqrt[4]{(-2)^4} = \sqrt[4]{2^4} = 2$)

Ex3) Let's simplify the following radical expressions:

a. $\sqrt[3]{-81} = \sqrt[3]{-1 \cdot 81} = \sqrt[3]{-1 \cdot 3^4} = \sqrt[3]{(-1)^3 \cdot 3^3 \cdot 3} = -3\sqrt[3]{3}$

You Try!

b. $\sqrt{x^{10}} = \sqrt{\qquad \cdot \qquad} = \sqrt{(\qquad)^2} = |\qquad|$

c. $\sqrt[4]{48y^6} =$

d. $\dfrac{3+\sqrt{81a^2}}{3} =$

Is $\sqrt{a+b}$ equal to $\sqrt{a} + \sqrt{b}$?

Since,

$$\sqrt{9+16} = \sqrt{25} = 5$$

$$\sqrt{9} + \sqrt{16} = 3 + 4 = 7$$

Therefore, $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$.

Likewise, $\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$.

How can we rewrite the radical expression with rational exponents?

If $\sqrt[n]{a}$ represents a real number and $\frac{m}{n}$ is a positive rational number reduced to lowest terms, and $n \geq 2$ is an integer, then

$$\sqrt[n]{a} = a^{\frac{1}{n}}.$$

$$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}.$$

Ex4) Rewrite radical expressions with rational exponents and simplify:

a. $\sqrt[3]{1000} = (1000)^{\frac{1}{3}} = (10^3)^{\frac{1}{3}} = 10^{3\cdot\left(\frac{1}{3}\right)} = 10$

You Try!

b. $\left(\sqrt[4]{5xy}\right)^9 =$

c. $\sqrt{16^3} =$

d. $\sqrt{\dfrac{125n^7}{169}} =$

Operations on Radical Expressions

Addition/Subtraction: Only can combine "like" radicals.

Ex5) Add or subtract the radical expressions as indicated and simplify.

a. $5\sqrt{2} - 2\sqrt{2} = (5-2)\sqrt{2} = 3\sqrt{2}$

b. $4\sqrt{7} + 3\sqrt{11} - 2\sqrt{7} + 5 = 2\sqrt{7} + 3\sqrt{11} + 5$

You Try!

c. $3\sqrt{27} - 4\sqrt{2} + 5\sqrt{3} = 9\sqrt{3} - 4\sqrt{2} + 5\sqrt{3} =$

d. $4\sqrt{2} - 7\sqrt{25} + 2\sqrt{2} - 2\sqrt{8} =$

Operations on Radical Expressions

Product Rule for Radicals: The product of two n^{th} roots is the n^{th} root of the product of the radicands.

$$\sqrt[n]{ab} = \sqrt[n]{a}\cdot\sqrt[n]{b} \quad \text{and} \quad \sqrt[n]{a}\cdot\sqrt[n]{b} = \sqrt[n]{ab}$$

Ex6) Multiply the radical expressions as indicated and simplify.

a. $\sqrt{5}\cdot\sqrt{10} = \sqrt{50} = \sqrt{25\cdot 2} = \sqrt{25}\cdot\sqrt{2} = 5\sqrt{2}$

You Try!

b. $\sqrt[3]{7}\cdot\sqrt[3]{9x^6} = \sqrt[3]{7\cdot 9\cdot (x^2)^3} =$

c. $\sqrt{3}\left(2 - 5\sqrt{6}\right) =$

d. $\left(1+\sqrt{3}\right)^2 =$

e. $\left(3+\sqrt{7}\right)\left(3-\sqrt{7}\right) =$

Radical Conjugates

Radical expressions that involve the sum and difference of the same two terms are called conjugates.

$$\sqrt{a}+\sqrt{b} \ \text{ and } \ \sqrt{a}-\sqrt{b} \text{ are conjugates of each other.}$$

The product of radical conjugates does not contain a radical.

$$\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2=a-b$$

For example, $\sqrt{2}+3\sqrt{6}$ and $\sqrt{2}-3\sqrt{6}$ are conjugates of each other. And their product is,

$$\left(\sqrt{2}+3\sqrt{6}\right)\left(\sqrt{2}-3\sqrt{6}\right)=\left(\sqrt{2}\right)^2-\sqrt{2}\left(3\sqrt{6}\right)+\sqrt{2}\left(3\sqrt{6}\right)-\left(3\sqrt{6}\right)^2=2-9\cdot 6=-52$$

Ex7) Determine the radical conjugate and find the product of conjugates.

a. $5-\sqrt{7}$ Radical Conjugate: $5+\sqrt{7}$

Product: $\left(5-\sqrt{7}\right)\left(5+\sqrt{7}\right)=5^2-\left(\sqrt{7}\right)^2=25-7=18$

You Try!

b. $3\sqrt{2}+4$ Radical Conjugate:

Product:

c. $\sqrt{11}-2\sqrt{3}$ Radical Conjugate:

Product:

Rationalizing Denominators

The process involving rewriting a radical expression as an equivalent expression in which the denominator no longer contains any radicals.

Ex8) Rationalize each denominator and simplify.

a. $\frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{3\cdot 7}}{\left(\sqrt{7}\right)^2} = \frac{\sqrt{21}}{7}$

<u>To remove the radical in the denominator</u>, multiplied the numerator and the denominator by $\sqrt{7}$.

b. $\sqrt[3]{\frac{7}{25}} = \sqrt[3]{\frac{7}{5^2}} = \frac{\sqrt[3]{7}}{\sqrt[3]{5^2}} \cdot \frac{\quad}{\quad} = \frac{\sqrt[3]{7\cdot 5}}{\sqrt[3]{5^3}} = \frac{\sqrt[3]{35}}{5}$

c. $\frac{2}{2+\sqrt{8}} = \frac{2}{2+\sqrt{8}} \cdot \frac{2-\sqrt{8}}{2-\sqrt{8}} = \frac{2(2-2\sqrt{2})}{4-8} = \frac{4(1-\sqrt{2})}{-4} =$

You Try!

d. $\frac{5}{\sqrt{50}} =$

e. $\frac{12}{\sqrt{7}+\sqrt{3}} =$

<u>Answer Key</u>

1. a. 8

   b. $\frac{1}{9}$

   c. -0.2

   d. $-x$

2. c. $3a^2b^2\sqrt[4]{a}$

   d. $\dfrac{2cd\sqrt[3]{d^2}}{5}$

3. b. $|x^5|$

   c. $2y\sqrt[4]{3y^2}$

   d. $1+3|a|$

4. b. $(5xy)^{\frac{9}{4}}$

   c. 2^6

   d. $\dfrac{5n^3\sqrt{5n}}{13}$

5. c. $14\sqrt{3}-4\sqrt{2}$

   d. $2\sqrt{2}-35$

6. b. $x^2\sqrt[3]{63}$

   c. $2\sqrt{3}-15\sqrt{2}$

   d. $4+2\sqrt{3}$

   e. 2

7. b. Radical Conjugate: $3\sqrt{2}-4$ Product: $(3\sqrt{2}+4)(3\sqrt{2}-4)=2$

   c. Radical Conjugate: $\sqrt{11}+2\sqrt{3}$ Product: $(\sqrt{11}-2\sqrt{3})(\sqrt{11}+2\sqrt{3})=-1$

8. c. $-1+\sqrt{2}$

   d. $\dfrac{\sqrt{2}}{2}$

   e. $3(\sqrt{7}-\sqrt{3})$