## I. Scientific Notation: Refresher

## Scientific notation helps us write very large and very small numbers.

## Scientific Notation to Numbers

Scientific Notation involves moving decimals.
$1.5 \times 10^{4}$
$=1.5 \underbrace{000} \quad \begin{array}{l}\begin{array}{l}\text { Because the exponent is Positive } \\ 4, \text { move the decimal point } 4 \text { places } \\ \text { to the right. } \\ \text { Add in Zeroes to fill the empty gaps. }\end{array} \\ =15000 \checkmark\end{array})$.

| $5.8 \times 10^{-4}$ | B | $3.76 \times 10^{-3}$ |
| :---: | :---: | :---: |
| $0005.8$ | 4, move the decimal point 4 places to the left. | 0003.7 |
| 0.00058 | Add in Zeroes to fill the empty gaps. | $=0.00376$ |

## Scientific Notation to Numbers

Scientific Notation involves moving decimals.

Because the exponent is Positive 5 , move the decimal point 5 places to the right. Add in Zeroes to fill the empty gaps.

Because the exponent is a Negative 3, move the decimal point 3 places to the left. Add in Zeroes to fill the empty gaps.

You Try!

1) $7.36 \times 10^{-6}=$ $\qquad$ 3) $8.2 \times 10^{10}=$ $\qquad$
2) $3.67 \times 10^{3}=$ $\qquad$ 4) $9.0 \times 10^{-4}=$ $\qquad$

Numbers into Scientific Notation
Numbers into Scientific Notation
0.0043

The Number is a decimal less than 1 , so the Exponent will be Negative.
$=1 \underbrace{5000}_{\text {4places }}$
Move the Decimal point to the LEFT to create a number between 1 and 10.
$=1.50$ O $\sigma$ Remove Zeroes that are not needed. $=\boldsymbol{0} 004.3$
$=0 . \underbrace{0,0,3}_{3 \text { places }}$
Move the Decimal point to the RIGHT to create a number between 1 and 10 .

Remove Zeroes that are not needed.
$=1.5 \times 10^{4} \checkmark$
We moved 4 places so
$=4.3 \times 10^{-3}$
We moved 3 places so
Power of 10 is three : $10^{-\mathbf{3}}$

## You Try!

5) $86910000=$
6) $794100=$ $\qquad$
7) $0.0000000000321=$ $\qquad$ 8) $0.0035=$ $\qquad$

## II. Scientific Notation: Calculators

When entering scientific notation into a calculator, use the 'EE' button, NOT the $10^{x}$ button. Also note that for many calculators the 'EE' button requires the shift button to be pressed first.

When displaying scientific or engineering notation, most calculators us ' E ' instead of $\mathbf{X 1 0}$
For example: most calculators would display $3.9 \times 10^{-5}$ as $3.9 \mathrm{E}-5$
most calculators would display $7.55 \times 10^{19}$ as 7.55 E 19

## You Try!

To enter $8.6 \times 10^{-3}$ into my calculator, I would do the following:
Type ' $8.6^{\prime}$
Hit the 'EE' key
Type '-3'
Your calculator should display: 0.0086 or $8.6 \mathrm{E}-3$
Let's try the following multiplication problem $\left(3.2 \times 10^{3}\right)\left(7.3 \times 10^{4}\right)$
To enter it into the calculator, do the following:
Type '3.2'
Hit the 'EE' key
Type '3'


Now type the multiply button
Type '7.3'
Hit the 'EE' key
Type '4'


Now hit the enter/equals button
Your calculator should display the answer:
233600000 (if your calculator is in Scientific Notation Mode)
2.336E8 (if your calculator is in Scientific Notation Mode)
233.6E6 (if your calculator is in Engineering Notation Mode)

## For calculator practice, work all the problems in this handout using your calculator!

## III. Scientific Notation: Adding and Subtracting

To add and subtract numbers in scientific notation, first convert them to standard form, then do the operation, and finally convert them back to scientific notation.

| $8.2 \times 10^{4}+4.32 \times 10^{3}$ | Original Problem |
| :--- | :--- |
| $8.2 \times 10^{4}=82000$ | Convert from <br> scientific notation to <br> standard form |
| $8.32 \times 10^{3}=4320$ | Do the indicated <br> operation |
| $82000+4320=86320$ |  |
| $86320 \rightarrow 8.632 \times 10^{4}$ | Convert back into <br> scientific notation. |
| $8.2 \times 10^{4}+4.32 \times 10^{3}=$ | Final Answer |
| $8.632 \times 10^{4}$ |  |


| $3.4 \times 10^{-4}-3.4 \times 10^{-3}$ | Original Problem |
| :--- | :--- |
| $3.4 \times 10^{-4}=0.00034$ | Convert from <br> scientific notation to <br> standard form |
| $0.4 \times 10^{-3}=0.0034$ | Do the indicated <br> operation |
| $-0.00306 \rightarrow-3.06 \times 10^{-3}$ | Convert back into <br> scientific notation. |
| $3.4 \times 10^{-4}-3.4 \times 10^{-3}=$ | Final Answer |
| $-3.06 \times 10^{-3}$ |  |

## You Try!

1) $8.54 \times 10^{-6}-1.2 \times 10^{-7}=$ $\qquad$
2) $7.88 \times 10^{-6}+2.222 \times 10^{-4}=$ $\qquad$
3) $5.6 \times 10^{4}+6.782 \times 10^{3}=$ $\qquad$ 4) $-7.6 \times 10^{4}-2.7 \times 10^{3}=$ $\qquad$

## IV. Scientific Notation: Multiplication and Division

To multiply and divide numbers in scientific notation, break the problem up between the coefficients/constants and the exponents, always remember to check that your final answer is in proper standard notation form!

| $\left(3.2 \times 10^{3}\right)\left(7.4 \times 10^{2}\right)$ | Original Problem |
| :---: | :---: |
| $(3.2 \cdot 7.4)\left(10^{3} \cdot 10^{2}\right)$ | Use the commutative property of multiplication to rewrite the problem with the constants together and powers logether. |
|  | Multiply the constants. Since your powers have like bases, you can add the exponents. <br> This is not in proper form. I need to move the decimall |
| $23.68 \times 10^{5}$ <br> From here I move the decimal 5 spaces to the right. <br> If I move the decimal back one spot, then I would need to move it one extra space to the right, so change the exponent from 5 to 6 . $2.368 \times 10^{6}$ | I have to move the decimal back one space so that it is placed after the ones number. <br> Change the exponent to a 6 I |
| $\begin{gathered} \left(3.2 \times 10^{3}\right)\left(7.4 \times 10^{2}\right)= \\ 2.368 \times 10^{6} \end{gathered}$ | Final Answer! |


| $\frac{3 \times 10^{-2}}{4 \times 10^{5}}$ | Original Problem |
| :---: | :---: |
| $\begin{aligned} & \frac{3 \times 10^{-2}}{4 \times 10^{5}} \\ & .75 \end{aligned}$ | Step 1: Divide the coefficients. $(3 / 4=.75)$ |
| $\begin{aligned} & \frac{3 \times 10^{-2}}{4 \times 10^{5}}= \\ & .75 \times 10^{-7} \end{aligned}$ | Step 2: Subtract the exponents of like bases. ( $-2-5=-7$ ) |
| $\begin{aligned} & .75 \times 10^{-7} \\ & 7.5 \times 10^{-8} \end{aligned}$ | Step 3: 75 is not written in correct scientific notation. It must read 7.5 , so we need to move the decimal to the right one space. Therefore, you have to change the exponent to -8 , because you'll have to move one more space to the left! |
| $\begin{aligned} & \frac{3 \times 10^{-2}}{4 \times 10^{5}}= \\ & 7.5 \times 10^{-8} \end{aligned}$ | Final Answer. |

## You Try!

1) $3.4 \times 10^{-6}\left(2.0 \times 10^{-7}\right)=$ $\qquad$
2) $8.8 \times 10^{-6} / 4.0 \times 10^{-3}=$ $\qquad$
3) $-6.6 \times 10^{-6} / 3.0 \times 10^{3}=$ $\qquad$ 5) $-8.4 \times 10^{4}\left(3.7 \times 10^{3}\right)=$ $\qquad$
4) $6.12 \times 10^{99} / 3.0 \times 10^{98}=$ $\qquad$ 6) $4.8 \times 10^{8}\left(3.9 \times 10^{-2}\right)=$ $\qquad$

## V. Rounding and Significant Figures

## When translating numbers into scientific notation, one often rounds.

For example,
The speed of light in a vacuum is approximately 186,282 miles/second
We could write this in scientific notation as $1.86282 \times 10^{5} \mathrm{miles} /$ second but this defeats the purpose of scientific notation-to make large and small numbers more manageable.
So, we often round when we write in scientific notaltion, we would say the speed of light in a vacuum is approximately $2 \times 10^{5} \mathrm{miles} /$ second.
NOTICE we could round this number in many different ways:
$2 \times 10^{5} \mathrm{miles} / \mathrm{sec}$ ond
$1.9 \times 10^{5} \mathrm{miles} / \mathrm{second}$
$1.86 \times 10^{5} \mathrm{miles} /$ second
$1.863 \times 10^{5} \mathrm{miles} /$ second

So how do we know at which decimal place to round the number?
Often the problem will tell you. The instructions will indicate how to round off the number.
REMEMBER: ALWAYS ROUND AS THE LAST STEP OF SOLVING THE PROBLEM

## You Try!

The gravitational constant is $6.67408 \times 10^{-11}$

1) Round to 3 decimals: $\qquad$
2) Round to the nearest whole number: $\qquad$
3) Round to 1 decimal: $\qquad$
Now try these:
4) $9.81 \times 10^{-6} / 4.7 \times 10^{-3}$ (round to three decimals) $\qquad$
5) $-6.68 \times 10^{-6} / 7.631 \times 10^{3}$ (round to six decimals) $\qquad$
6) $9.247 \times 10^{6} / 4.5 \times 10^{5}$ (round to the nearest whole number) $\qquad$

So how do we know at which decimal place to round the number?
More often, you will be told/know how many significant figures you should round to.

## Significant Figures: Quick Review

| Rule | Example |
| :--- | :---: |
| 1. All non-zeroes are significant | $\underline{2.25}$ (3 significant figures) |
| 2. Leading zeroes are NOT significant | 0.00000034 (2 significant figures) |

## 3. Trailing zeroes are significant ONLY if an explicit decimal point is present

4. Trapped zeroes are significant

200 (1 significant figure)
200. (3 significant figures)
2.00 (3 significant figures)
0.0509 ( 3 significant figures) 2045 (4 significant figures)

## Always ask your instructor, but in general...

- Round as the LAST STEP of the calculation
- For multiplication and division, least number of significant figures in any number determines the number of significant figures in the answer.

You Try!
378002.0030

Has many significant figures? $\qquad$
. 00003420
Has many significant figures? $\qquad$

Written in scientific notation: $\qquad$ Written in scientific notation: $\qquad$
0.000000502

Has many significant figures? $\qquad$
-1.003
Has many significant figures? $\qquad$
Written in scientific notation: $\qquad$ Written in scientific notation: $\qquad$
378002.0030 ( 0.000000502 )

Answer will have how many significant figures?
-1.003 / . 00003420
Answer will have how many significant figures?

Answer written in scientific notation:
Answer written in scientific notation:

## VI. Engineering Notation: A Variant

Engineering notation is a special form of scientific notation, where the exponent of the ten is always a multiple of three. To accomplish this, the decimal is moved as needed.

| Number | In Scientific <br> Notation | In Engincering <br> Notation | Explanation |
| :--- | :--- | :--- | :--- |
| 84200000 | $8.42 \times 10^{7}$ | $84.2 \times 10^{6}$ | Move decimal point to the right one place so <br> that $10^{7}$ becomes $10^{6}$ to make the exponent a <br> multiple of three |
| .000039 | $3.90 \times 10^{-4}$ | $390 \times 10^{-6}$ | Move decimal point to the right two spaces so <br> that $10^{-5}$ becomes $10^{-6}$ to make the exponent a <br> multiple of three |
| 2400 | $2.4 \times 10^{-3}$ | $2.4 \times 10^{-3}$ | No change because $10^{-3}$ is already a multiple <br> of three. |

You Try!

980,000,000,000
-0.000078
In scientific notation: $\qquad$

In engineering notation: $\qquad$

Most calculators can display in either scientific or engineering notation. Consult your manuals, but the option is typically under "mode" 'NORMAL' for regular notation
'SCI' for scientific notation 'ENG' for engineering notation

One of the benefits of engineering notation is that one can more easily speak about large or small numbers by using the following prefixes.

| Value | Prefix | Symbol |
| :---: | :---: | :---: |
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{-3}$ | milji | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |

For example,
Instead of saying a retaining wall holds back $4.0 \times 10^{7}$ liters of water, one could use engineering notation: 40 mega liters of water ( $40 \times 10^{6}$ liters).

Instead of saying a $7.3 \times 10^{13}$ byte hard drive, one could use engineering notation: a 73 terabyte hard drive ( $73 \times 10^{12}$ bytes)

Instead of saying a $6.1 \times 10^{-7}$ meter wavelength, one could use engineering notation: a 610 nanometer wavelength ( $610 \times 10^{-9}$ )
I. Scientific Notation Refresher

1) 0.000000736
2) 3760
3) 82000000000
4) 0.00090
5) $8.691 \times 10^{7}$
6) $3.21 \times 10^{-11}$
7) $7.941 \times 10^{5}$
8) $3.5 \times 10^{-3}$
III. Adding and Subtracting
9) $8.42 \times 10^{-6}$
10) $6.2782 \times 10^{4}$
11) $2.3008 \times 10^{-4}$
12) $-7.87 \times 10^{4}$
IV. Multiplication and Division
13) $6.8 \times 10^{-13}$
14) $-2.2 \times 10^{-9}$
15) $2.04 \times 10^{1}$
16) $2.2 \times 10^{-3}$
17) $-3.108 \times 10^{8}$
18) $1.872 \times 10^{7}$

## V. Rounding and Significant Figures

1) $6.674 \times 10^{-11}$
2) $7 \times 10^{-11}$
3) $6.7 \times 10^{-11}$
4) $2.087 \times 10^{-3}$
5) $-8.753768 \times 10^{-10}$
6) $2 \times 10^{1}$

## Significant Figures: Quick Review

### 378002.0030

$$
\text { How many sig figs? } 10
$$

Written in scientific notation: $\quad 3.780020030 \times 10^{5}$
0.000000502

How many sig figs? 3
Written in scientific notation: $\quad 5.02 \times 10^{-7}$ 378002.0030(0.000000502)

How many sig figs? 3
Answer:
. 00003420
How many sig figs?
4
Written in scientific notation: $\quad 3.420 \times 10^{-5}$
-1.003
How many sig figs? 4
Written in scientific notation: $-1.003 \times 10^{1}$
-1.003 / . 00003420
How many sig figs?
4
Answer:
VI. Engineering Notation

980,000,000,000
$9.8 \times 10^{11}$
$980 \times 10^{9}$
0.000078
$-7.8 \times 10^{-5}$
$-78 \times 10^{-6}$

